MIT 6.035
Top-Down Parsing

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Orientation

- Language specification
  - Lexical structure – regular expressions
  - Syntactic structure – grammar

  This Lecture - recursive descent parsers
  - Code parser as set of mutually recursive procedures
  - Structure of program matches structure of grammar

Starting Point

- Assume lexical analysis has produced a sequence of tokens
  - Each token has a type and value
  - Types correspond to terminals
  - Values to contents of token read in
- Examples
  - Int 549 – integer token with value 549 read in
  - if - if keyword, no need for a value
  - AddOp + - add operator, value +

Basic Approach

- Start with Start symbol
- Build a leftmost derivation
  - If leftmost symbol is nonterminal, choose a production and apply it
  - If leftmost symbol is terminal, match against input
  - If all terminals match, have found a parse!
  - Key: find correct productions for nonterminals

Graphical Illustration of Leftmost Derivation

Sentential Form

$NT_1\ T_1\ T_2\ T_3\ NT_2\ NT_3$

Apply Production Here

Not Here

Grammar for Parsing Example

$Start \rightarrow Expr$

$Expr \rightarrow Expr + Term$

$Expr \rightarrow Expr - Term$

$Expr \rightarrow Term$

$Term \rightarrow Term * \text{Int}$

$Term \rightarrow Term / \text{Int}$

$Term \rightarrow \text{Int}$

- Set of tokens is
  { +, -, *, /, Int }, where
  Int = [0-9][0-9]*

  For convenience, may represent each Int n token by n

Parsing Example

Parse Tree

Start

Current Position in Parse Tree

Remaining Input

2-2*2

Sentential Form

Start

Parsing Example

Parse Tree

Start

Expr

Current Position in Parse Tree

Remaining Input

2-2*2

Sentential Form

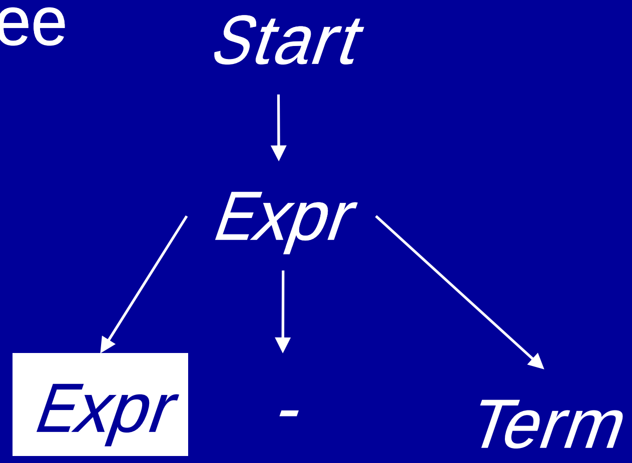
Expr

Applied Production

Start $\rightarrow$ *Expr*

Parsing Example

Parse Tree



Remaining Input

2-2*2

Sentential Form

Expr - Term

$Expr \rightarrow Expr + Term$

$\boxed{Expr \rightarrow Expr - Term}$

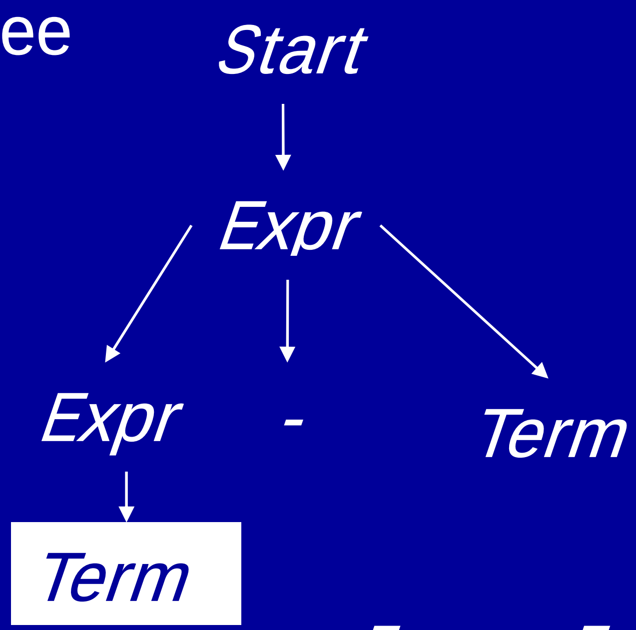
$Expr \rightarrow Term$

Applied Production

$Expr \rightarrow Expr - Term$

Parsing Example

Parse Tree



Expr $\rightarrow$ *Expr + Term*

Expr $\rightarrow$ *Expr - Term*

$\boxed{\textit{Expr} \rightarrow \textit{Term}}$

Remaining Input

2-2*2

Sentential Form

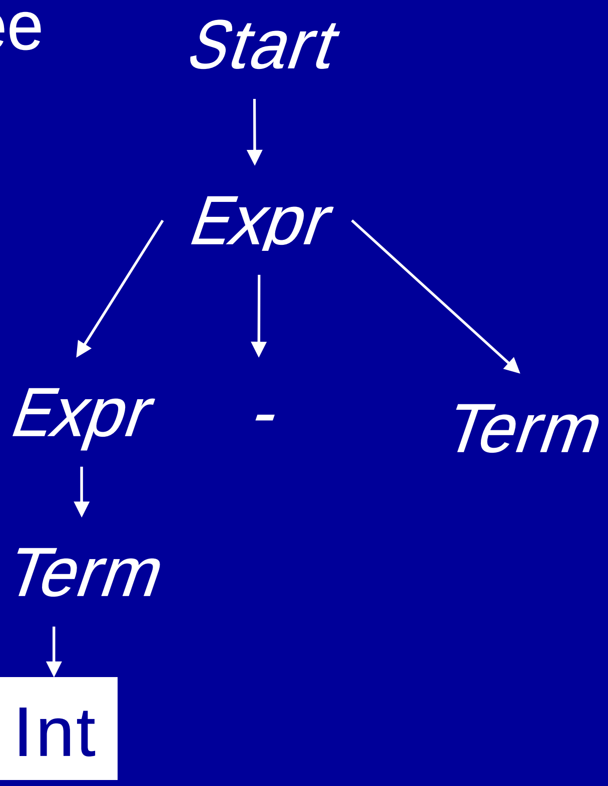
Term - Term

Applied Production

Expr $\rightarrow$ *Term*

Parsing Example

Parse Tree



Remaining Input

2-2*2

Sentential Form

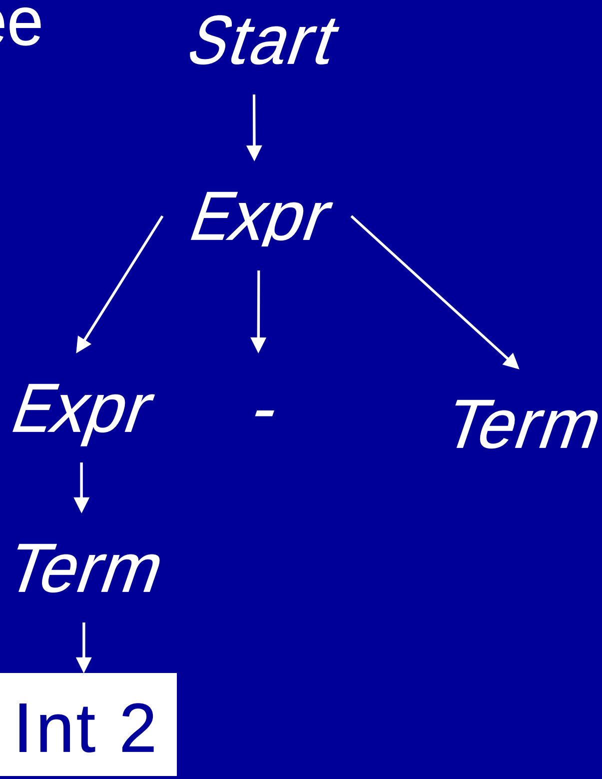
Int - *Term*

Applied Production

Term → Int

Parsing Example

Parse Tree



Match Input Token!

Remaining Input

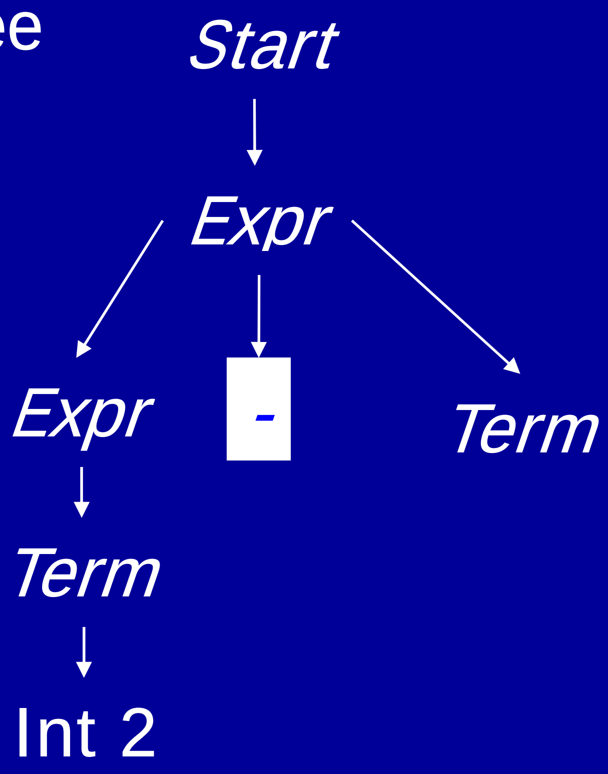
2-2*2

Sentential Form

2 - *Term*

Parsing Example

Parse Tree



Match Input Token!

Remaining Input

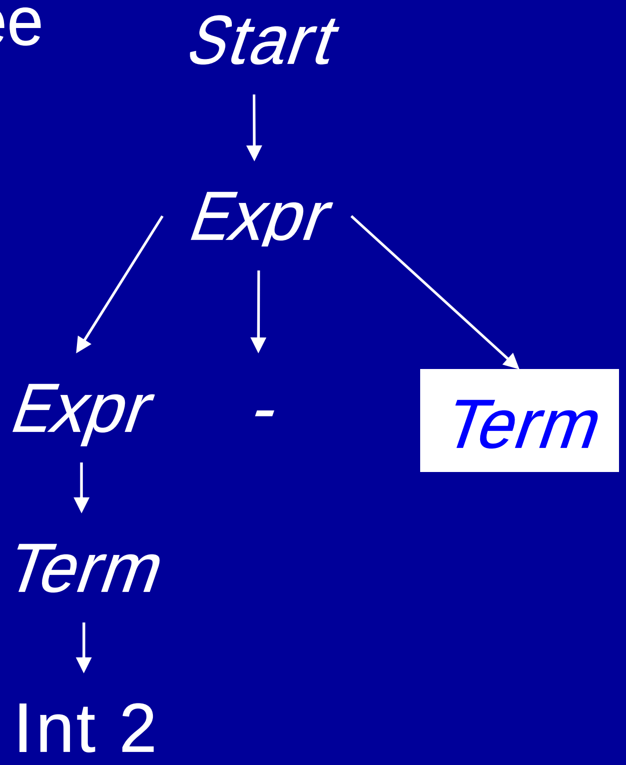
-2*2

Sentential Form

2 - *Term*

Parsing Example

Parse Tree



Match Input Token!

Remaining Input

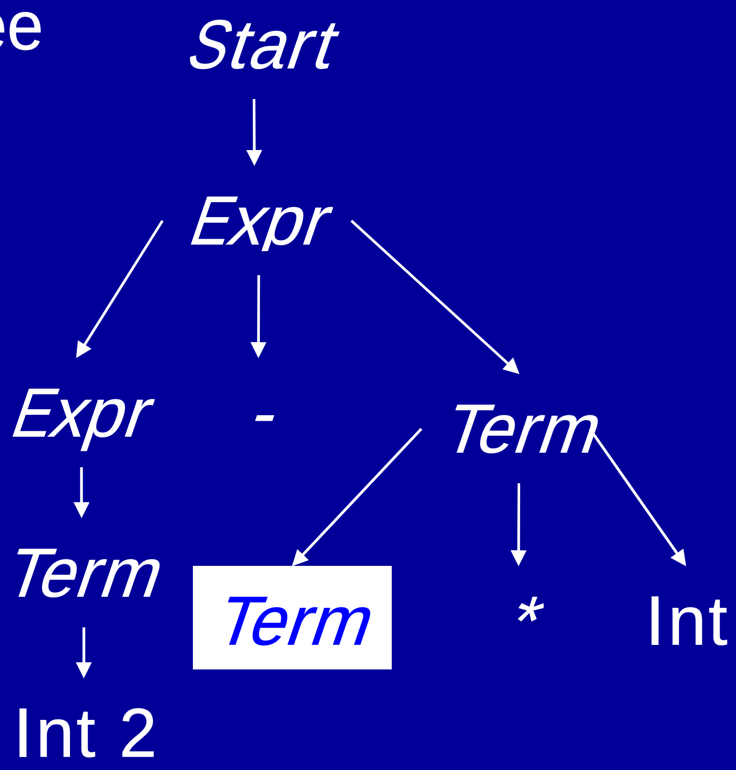
2*2

Sentential Form

2 - *Term*

Parsing Example

Parse Tree



Remaining Input

2*2

Sentential Form

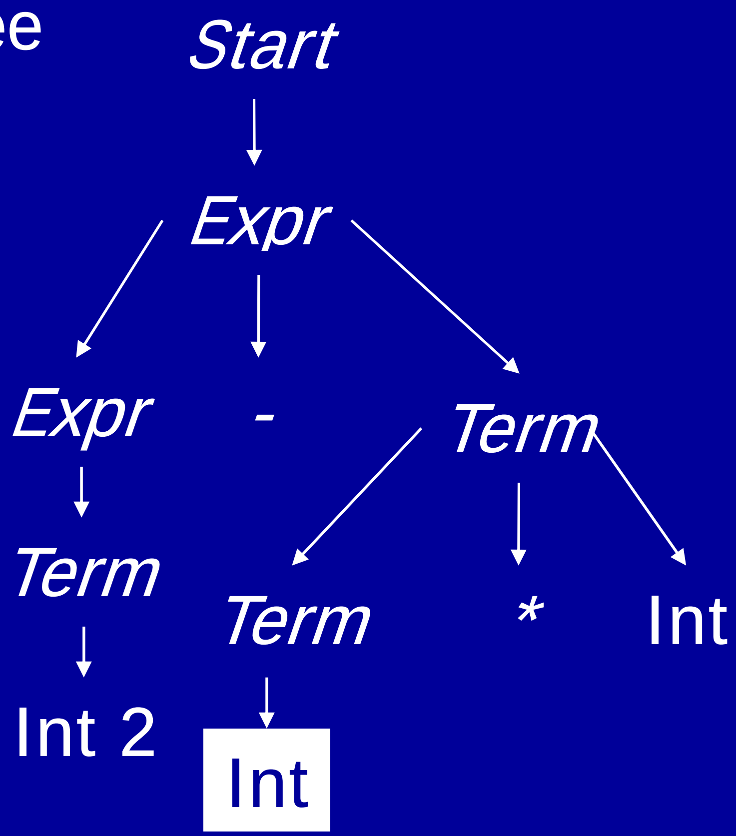
2 - *Term**Int

Applied Production

Term → *Term* * Int

Parsing Example

Parse Tree



Remaining Input

2*2

Sentential Form

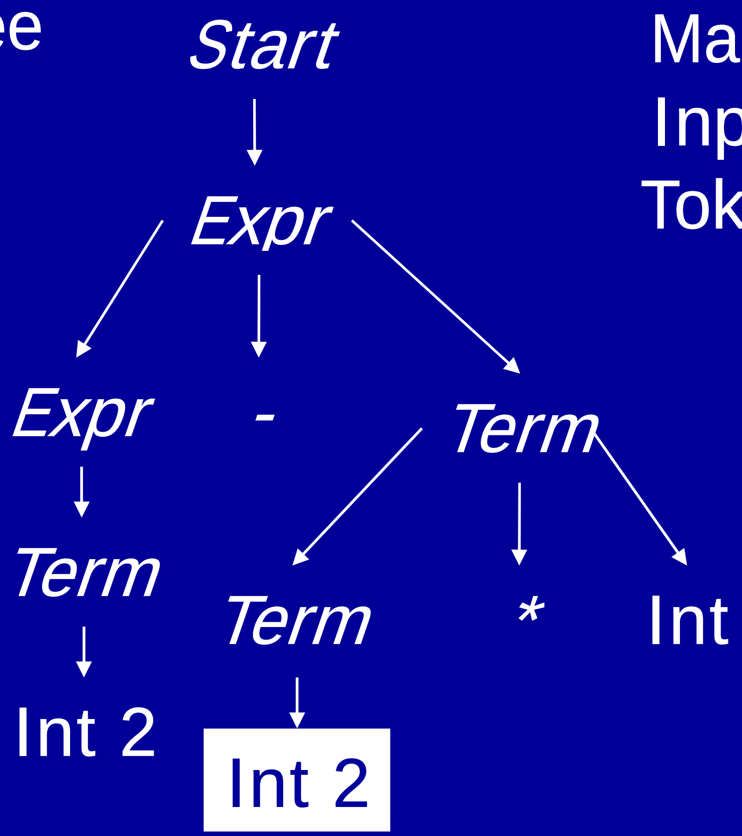
2 - Int * Int

Applied Production

Term → Int

Parsing Example

Parse Tree



Match Input Token!

Remaining Input

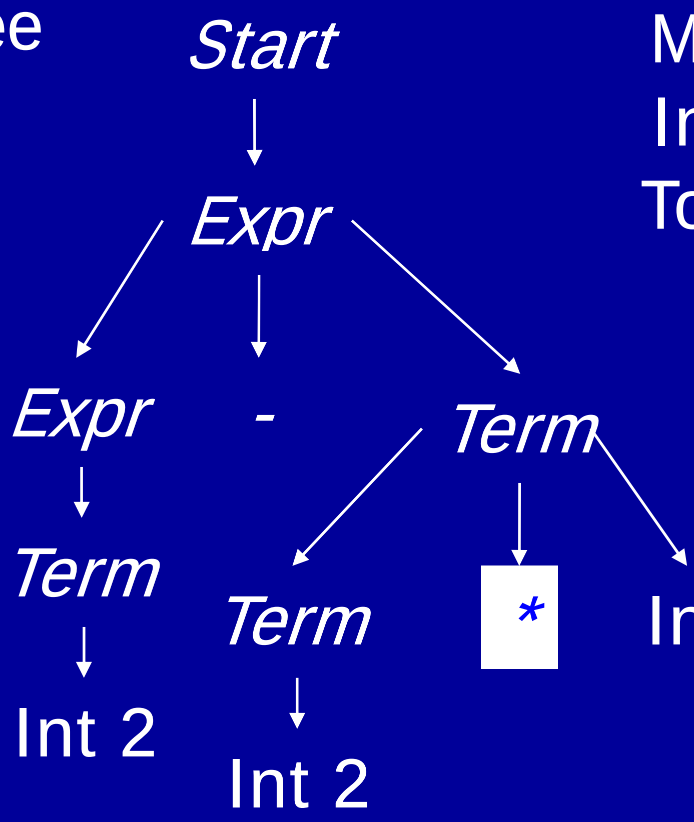
2*2

Sentential Form

2 - 2 * Int

Parsing Example

Parse Tree



Match Input Token!

Remaining Input

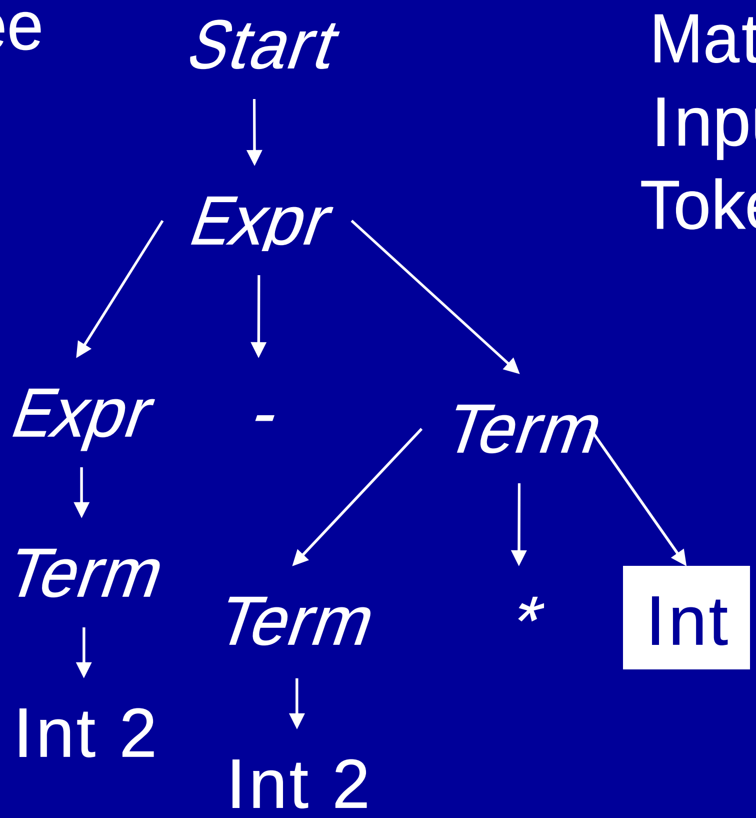
*2

Sentential Form

2 - 2 * Int

Parsing Example

Parse Tree



Match Input Token!

Remaining Input

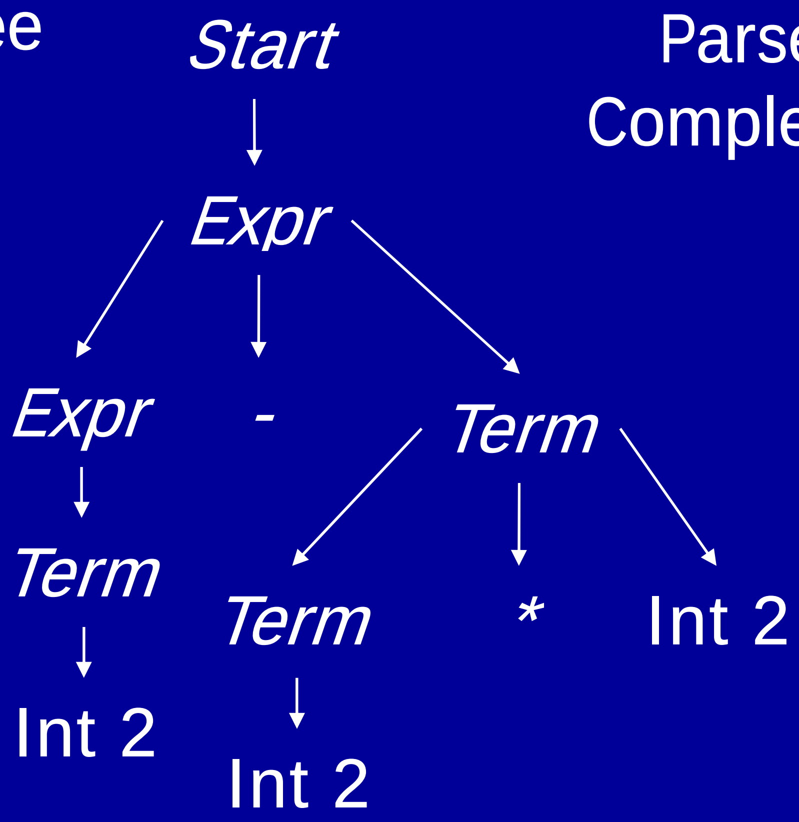
2

Sentential Form

2 - 2 * Int

Parsing Example

Parse Tree



Parse Complete!

Remaining Input

2

Sentential Form

2 - 2 *2

Summary

- Three Actions (Mechanisms)
  - Apply production to expand current nonterminal in parse tree
  - Match current terminal (consuming input)
  - Accept the parse as correct
- Parser generates preorder traversal of parse tree
  - visit parents before children
  - visit siblings from left to right

Policy Problem

- Which production to use for each nonterminal?
- Classical Separation of Policy and Mechanism
- One Approach: Backtracking
  - Treat it as a search problem
  - At each choice point, try next alternative
  - If it is clear that current try fails, go back to previous choice and try something different
- General technique for searching
- Used a lot in classical AI and natural language processing (parsing, speech recognition)

Backtracking Example

Parse Tree

Start

Remaining Input

2-2*2

Sentential Form

Start

Backtracking Example

Parse Tree

Start

↓

$\boxed{Expr}$

Remaining Input

2-2*2

Sentential Form

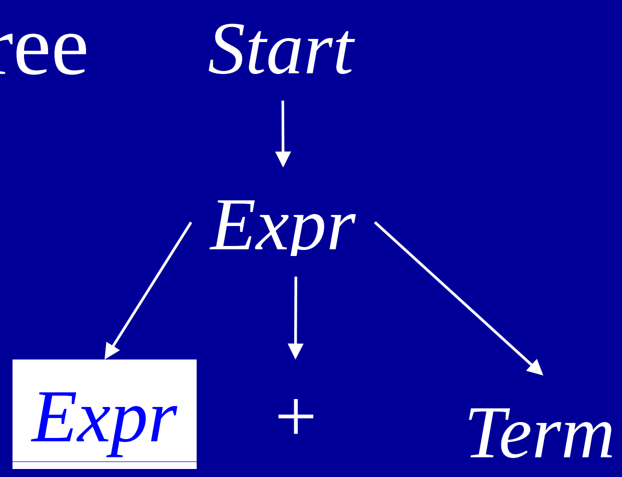
Expr

Applied Production

$Start \rightarrow Expr$

Backtracking Example

Parse Tree



Remaining Input

2-2*2

Sentential Form

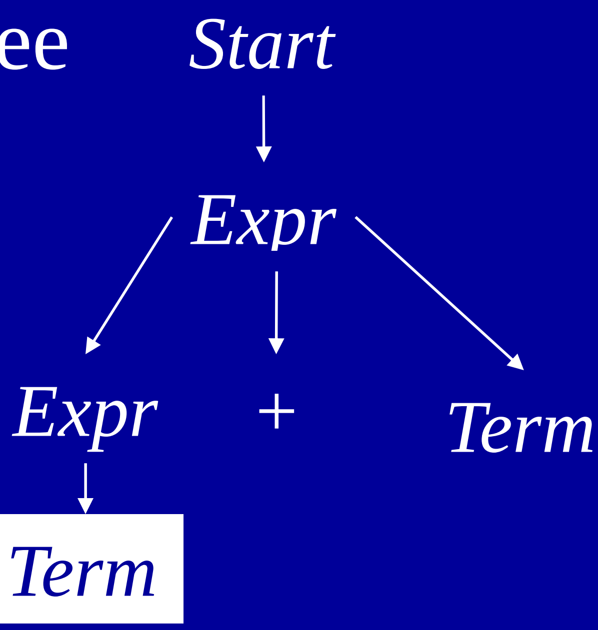
$Expr + Term$

Applied Production

$Expr \rightarrow Expr + Term$

Backtracking Example

Parse Tree



Remaining Input

2-2*2

Sentential Form

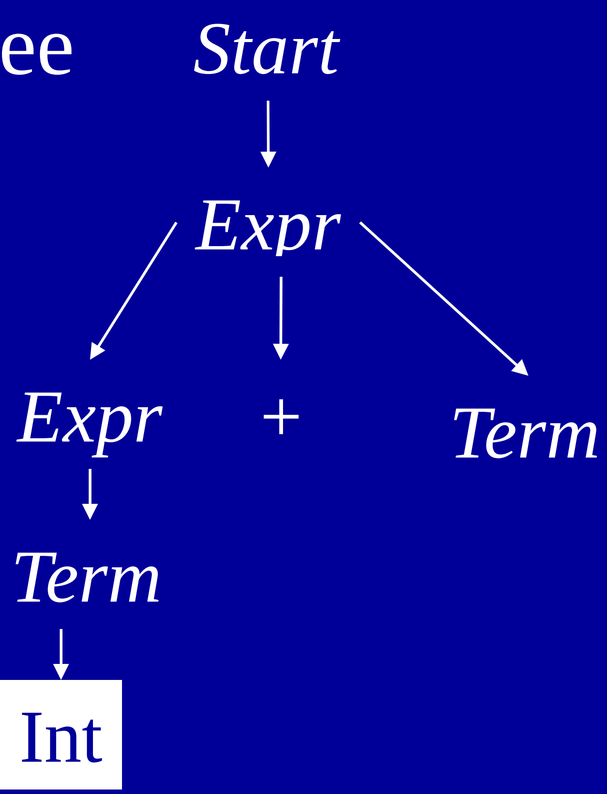
Term + *Term*

Applied Production

$Expr \rightarrow Term$

Backtracking Example

Parse Tree



Match Input Token!

Remaining Input

2-2*2

Sentential Form

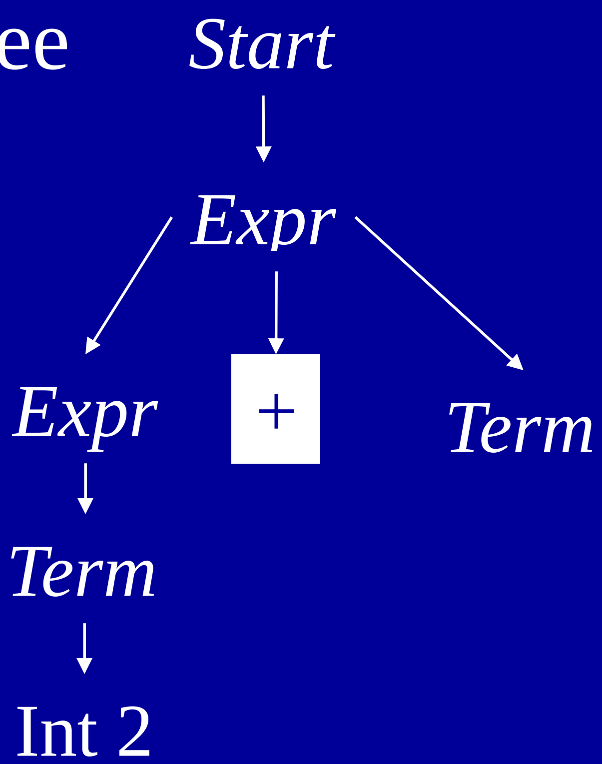
Int + *Term*

Applied Production

Term → Int

Backtracking Example

Parse Tree



Can't Match Input Token!

Remaining Input

-2*2

Sentential Form

2 - *Term*

Applied Production

Term $\rightarrow$ Int

Backtracking Example

Parse Tree

Start

↓

Expr

So Backtrack!

Remaining Input

2-2*2

Sentential Form

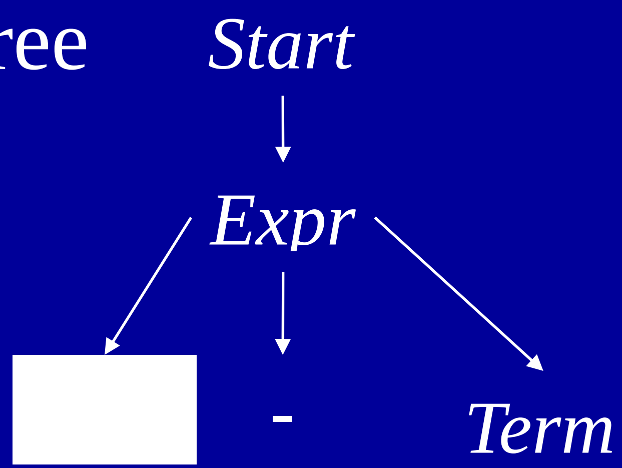
Expr

Applied Production

Start → *Expr*

Backtracking Example

Parse Tree



Remaining Input

2-2*2

Sentential Form

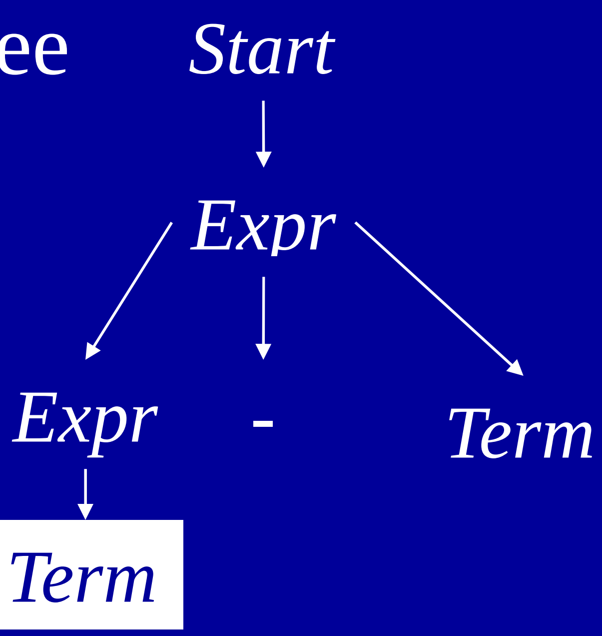
Expr - Term

Applied Production

$Expr \rightarrow Expr - Term$

Backtracking Example

Parse Tree



Remaining Input

2-2*2

Sentential Form

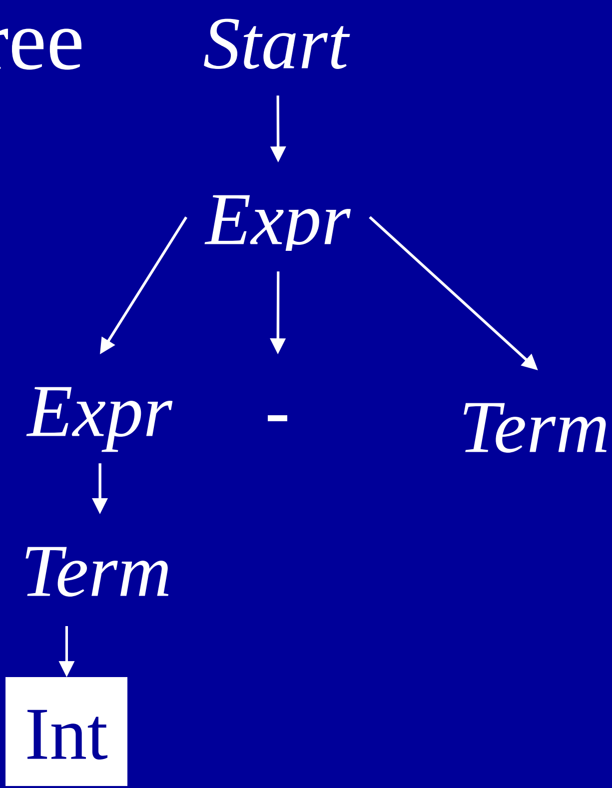
Term - *Term*

Applied Production

$Expr \rightarrow Term$

Backtracking Example

Parse Tree



Remaining Input

2-2*2

Sentential Form

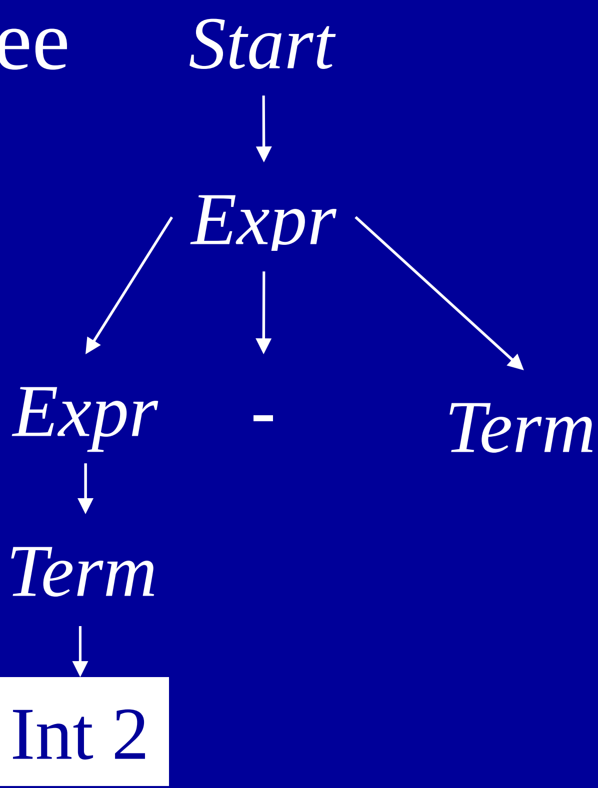
Int - *Term*

Applied Production

Term → Int

Backtracking Example

Parse Tree



Match Input Token!

Remaining Input

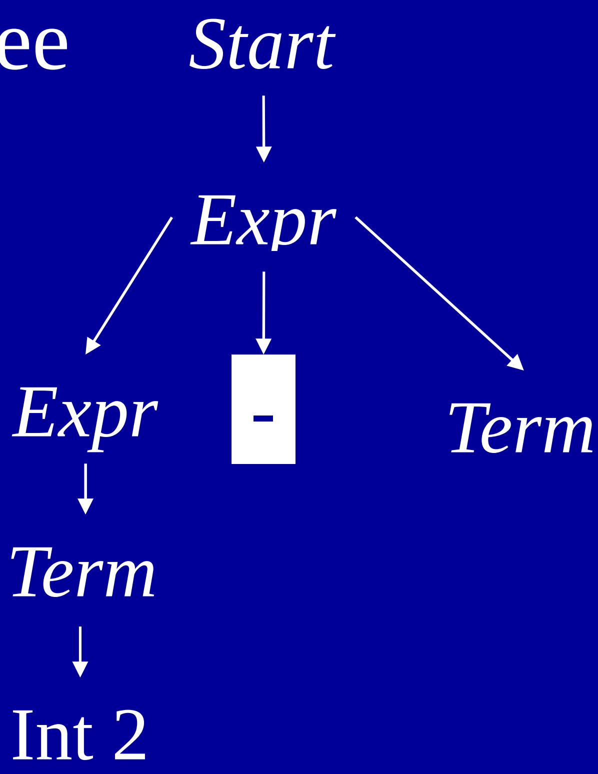
-2*2

Sentential Form

2 - *Term*

Backtracking Example

Parse Tree



Match Input Token!

Remaining Input

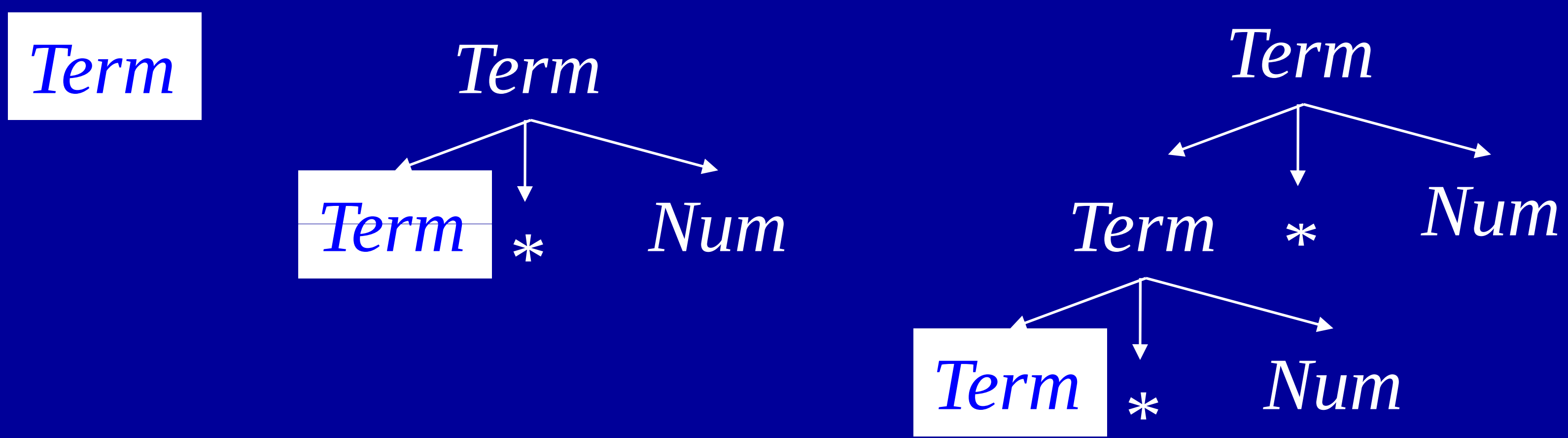
2*2

Sentential Form

2 - *Term*

Left Recursion + Top-Down Parsing = Infinite Loop

- Example Production: *Term* → *Term* Num*
- Potential parsing steps:



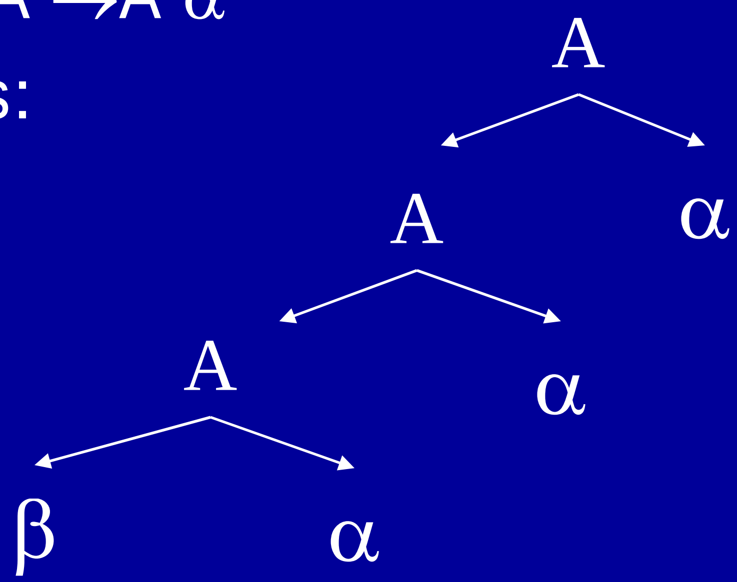
General Search Issues

- Three components
  - Search space (parse trees)
    Search algorithm (parsing algorithm)
  - Goal to find (parse tree for input program)
- Would like to (but can't always) ensure that
  - Find goal (hopefully quickly) if it exists
  - Search terminates if it does not
- Handled in various ways in various contexts
  - Finite search space makes it easy
  - Exploration strategies for infinite search space
  - Sometimes one goal more important (model checking)
- For parsing, hack grammar to remove left recursion

Eliminating Left Recursion

- Start with productions of form
  - $A \rightarrow A\ \alpha$
  - $A \rightarrow \beta$
  - α, β sequences of terminals and nonterminals that do not start with A
- Repeated application of $A \rightarrow A\ \alpha$

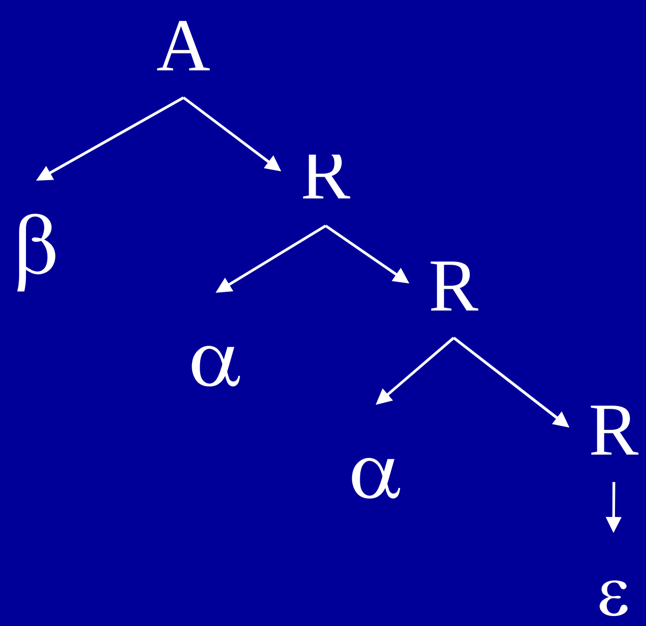
  builds parse tree like this:



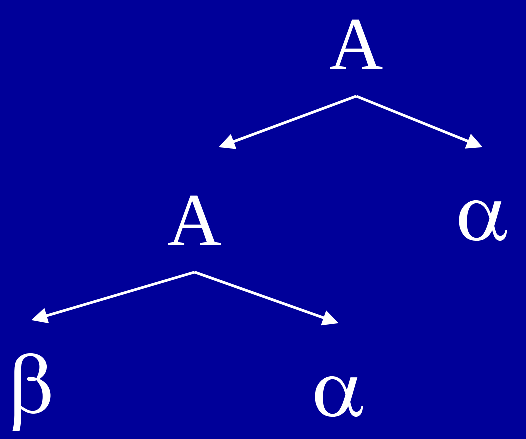
Eliminating Left Recursion

- Replacement productions
  - $A \rightarrow A\ \alpha$ $\qquad$ $A \rightarrow \beta\ R$ $\qquad$ R is a new nonterminal
  - $A \rightarrow \beta$ $\qquad$ $R \rightarrow \alpha\ R$
  - $\qquad$ $R \rightarrow \varepsilon$

New Parse Tree



Old Parse Tree



Hacked Grammar

Original Grammar Fragment

$Term \rightarrow Term$ * Int

$Term \rightarrow Term$ / Int

$Term \rightarrow$ Int

New Grammar Fragment

$Term \rightarrow$ Int $Term'$

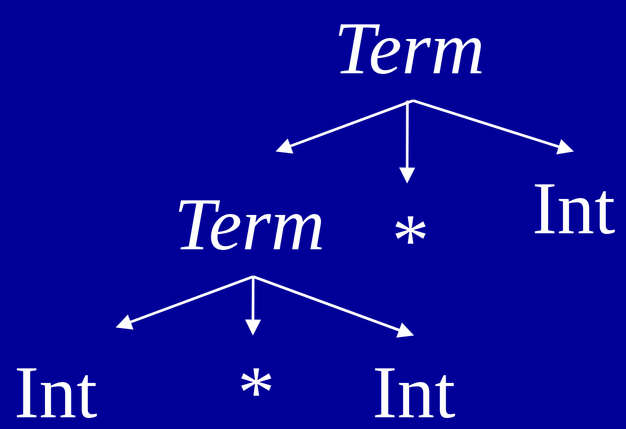
$Term' \rightarrow$ * Int $Term'$

$Term' \rightarrow$ / Int $Term'$

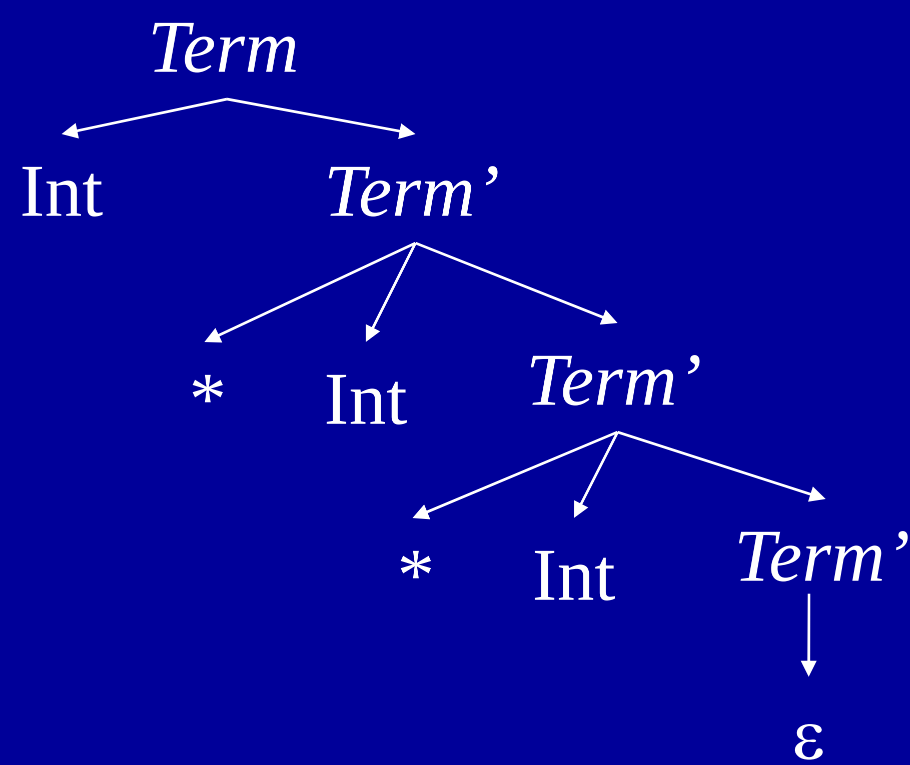
$Term' \rightarrow \varepsilon$

Parse Tree Comparisons

Original Grammar



New Grammar



Eliminating Left Recursion

- Changes search space exploration algorithm
  - Eliminates direct infinite recursion
  - But grammar less intuitive

Sets things up for predictive parsing

Predictive Parsing

- Alternative to backtracking
- Useful for programming languages, which can be designed to make parsing easier
- Basic idea
  - Look ahead in input stream
  - Decide which production to apply based on next tokens in input stream
  - We will use one token of lookahead

Predictive Parsing Example Grammar

Start → *Expr*

Expr → *Term Expr'*

Expr' → + *Expr'*

Expr' → - *Expr'*

Expr' → ε

Term → Int *Term'*

Term' → * Int *Term'*

Term' → / Int *Term'*

Term' → ε

Choice Points

- Assume *Term'* is current position in parse tree
- Have three possible productions to apply

  Term' $\rightarrow$ *** Int *Term'*

  Term' $\rightarrow$ */* Int *Term'*

  Term' $\rightarrow \varepsilon$
- Use next token to decide
  - If next token is *, apply *Term'* $\rightarrow$ *** Int *Term'*
  - If next token is /, apply *Term'* $\rightarrow$ */* Int *Term'*
  - Otherwise, apply *Term'* $\rightarrow \varepsilon$

Predictive Parsing + Hand Coding = Recursive Descent Parser

- One procedure per nonterminal *NT*
  - Productions $NT \rightarrow \beta_1$, ..., $NT \rightarrow \beta_n$
  - Procedure examines the current input symbol T to determine which production to apply
    - If $T \in First(\beta_k)$
    - Apply production k
    - Consume terminals in β_k (check for correct terminal)
    - Recursively call procedures for nonterminals in β_k
  - Current input symbol stored in global variable token
- Procedures return
  - true if parse succeeds
  - false if parse fails

Example

```
Boolean Term()
   if (token = Int n) token = NextToken(); return(TermPrime())
   else return(false)
Boolean TermPrime()
   if (token = *)
         token = NextToken();
         if (token = Int n) token = NextToken(); return(TermPrime())
         else return(false)
   else if (token = /)
         token = NextToken();
         if (token = Int n) token = NextToken(); return(TermPrime())
         else return(false)
   else return(true)
```

Term $\rightarrow$ Int *Term'*
Term' $\rightarrow$ * Int *Term'*
Term' $\rightarrow$ / Int *Term'*
Term' $\rightarrow$ ε

Multiple Productions With Same Prefix in RHS

- Example Grammar

  NT $\rightarrow$ if then

  NT $\rightarrow$ if then else

  Assume *NT* is current position in parse tree, and if is the next token
- Unclear which production to apply
  - Multiple k such that T $\in$ First(β_k)
  - if $\in$ First(if then)
  - if $\in$ First(if then else)

Solution: Left Factor the Grammar

- New Grammar Factors Common Prefix Into Single Production

  NT $\rightarrow$ if then NT'

  NT' $\rightarrow$ else

  NT' $\rightarrow$ ε
- No choice when next token is if!
- All choices have been unified in one production.

Nonterminals

- What about productions with nonterminals?

  $NT \to NT_1\ \alpha_1$

  $NT \to NT_2\ \alpha_2$

- Must choose based on possible first terminals that NT_1 and NT_2 can generate
- What if NT_1 or NT_2 can generate ε?
  - Must choose based on α_1 and α_2

NT derives ε

- Two rules
  - $NT \rightarrow \varepsilon$ implies NT derives ε
  - $NT \rightarrow NT_1 \ldots NT_n$ and for all $1 \leq i \leq n$ NT_i derives ε implies NT derives ε

Fixed Point Algorithm for Derives ε

for all nonterminals *NT*

  set *NT* derives ε to be false

for all productions of the form $NT \rightarrow \varepsilon$

  set *NT* derives ε to be true

while (some *NT* derives ε changed in last iteration)

  for all productions of the form $NT \rightarrow NT_1 \ldots NT_n$

    if (for all $1 \leq i \leq n$ NT_i derives ε)

      set *NT* derives ε to be true

First(β)

- T $\in$ First(β) if T can appear as the first symbol in a derivation starting from β
  1) $T \in$ First(T)
  2) First(S) $\subset$ First($S\,\beta$)
  3) NT derives ε implies First(β) $\subseteq$ First($NT\,\beta$)
  4) $NT \rightarrow S\,\beta$ implies First(S β) $\subseteq$ First(NT)

- Notation
  - T is a terminal, NT is a nonterminal, S is a terminal or nonterminal, and β is a sequence of terminals or nonterminals

Rules + Request Generate System of Subset Inclusion Constraints

Grammar

$Term' \to$ * Int $Term'$

$Term' \to$ / Int $Term'$

$Term' \to \varepsilon$

Rules

1) $T \in \mathrm{First}(T)$
2) $\mathrm{First}(S) \subseteq \mathrm{First}(S\,\beta)$
3) NT derives ε implies
   $\mathrm{First}(\beta) \subseteq \mathrm{First}(NT\,\beta)$
4) $NT \to S\,\beta$ implies
   $\mathrm{First}(S\,\beta) \subseteq \mathrm{First}(NT)$

Request: What is First($Term'$)?

Constraints

$\mathrm{First}(*\ Num\ Term') \subseteq \mathrm{First}(Term')$

$\mathrm{First}(/\ Num\ Term') \subseteq \mathrm{First}(Term')$

$\mathrm{First}(*) \subseteq \mathrm{First}(*\ Num\ Term')$

$\mathrm{First}(/) \subseteq \mathrm{First}(/\ Num\ Term')$

$* \in \mathrm{First}(*)$

$/ \in \mathrm{First}(/)$

Constraint Propagation Algorithm

Constraints

First(* *Num Term'*) $\subseteq$ First(*Term'*)

First(/ *Num Term'*) $\subseteq$ First(*Term'*)

First(*) $\subseteq$ First(* *Num Term'*)

First(/) $\subseteq$ First(/ *Num Term'*)

* $\in$ First(*)

/ $\in$ First(/)

Solution

First(*Term'*) = {}

First(* *Num Term'*) = {}

First(/ *Num Term'*) = {}

First(*) = {*}

First(/) − {/}

Initialize Sets to {}

Propagate Constraints Until Fixed Point

Constraint Propagation Algorithm

Constraints

First(** Num Term'*) $\subseteq$ First(*Term'*)

First(*/ Num Term'*) $\subseteq$ First(*Term'*)

First(*) $\subseteq$ First(** Num Term'*)

First(/) $\subseteq$ First(*/ Num Term'*)

* $\in$ First(*)

/ $\in$ First(/)

Solution

First(*Term'*) = {}

First(** Num Term'*) = {}

First(*/ Num Term'*) = {}

First(*) = {*}

First(/) – {/}

Grammar

Term' $\rightarrow$ *** Int *Term'*

Term' $\rightarrow$ */* Int *Term'*

Term' $\rightarrow$ ε

Constraint Propagation Algorithm

Constraints

First($*$ *Num Term'*) $\subseteq$ First(*Term'*)

First($/$ *Num Term'*) $\subseteq$ First(*Term'*)

First($*$) $\subseteq$ First($*$ *Num Term'*)

First($/$) $\subseteq$ First($/$ *Num Term'*)

$* \in$ First($*$)

$/ \in$ First($/$)

Solution

First(*Term'*) = {}

First($*$ *Num Term'*) = {$*$}

First($/$ *Num Term'*) = {/}

First($*$) = {$*$}

First($/$) = {/}

Grammar

Term' $\rightarrow$ $*$ Int *Term'*

Term' $\rightarrow$ $/$ Int *Term'*

Term' $\rightarrow$ ε

Constraint Propagation Algorithm

Constraints

First($*$ *Num Term'*) $\subseteq$ First(*Term'*)

First($/$ *Num Term'*) $\subseteq$ First(*Term'*)

First($*$) $\subseteq$ First($*$ *Num Term'*)

First($/$) $\subseteq$ First($/$ *Num Term'*)

$* \in$ First($*$)

$/ \in$ First($/$)

Solution

First(*Term'*) = {$*$, $/$}

First($*$ *Num Term'*) = {$*$}

First($/$ *Num Term'*) = {$/$}

First($*$) = {$*$}

First($/$) = {$/$}

Grammar

Term' $\rightarrow$ $*$ Int *Term'*

Term' $\rightarrow$ $/$ Int *Term'*

Term' $\rightarrow$ ε

Constraint Propagation Algorithm

Constraints

First(** Num Term'*) $\subseteq$ First(*Term'*)

First(*/ Num Term'*) $\subseteq$ First(*Term'*)

First(*) $\subseteq$ First(** Num Term'*)

First(/) $\subseteq$ First(*/ Num Term'*)

* $\in$ First(*)

/ $\in$ First(/)

Solution

First(*Term'*) = {*,/}

First(** Num Term'*) = {*}

First(*/ Num Term'*) = {/}

First(*) = {*}

First(/) = {/}

Grammar

Term' $\rightarrow$ *** Int *Term'*

Term' $\rightarrow$ */* Int *Term'*

Term' $\rightarrow$ ε

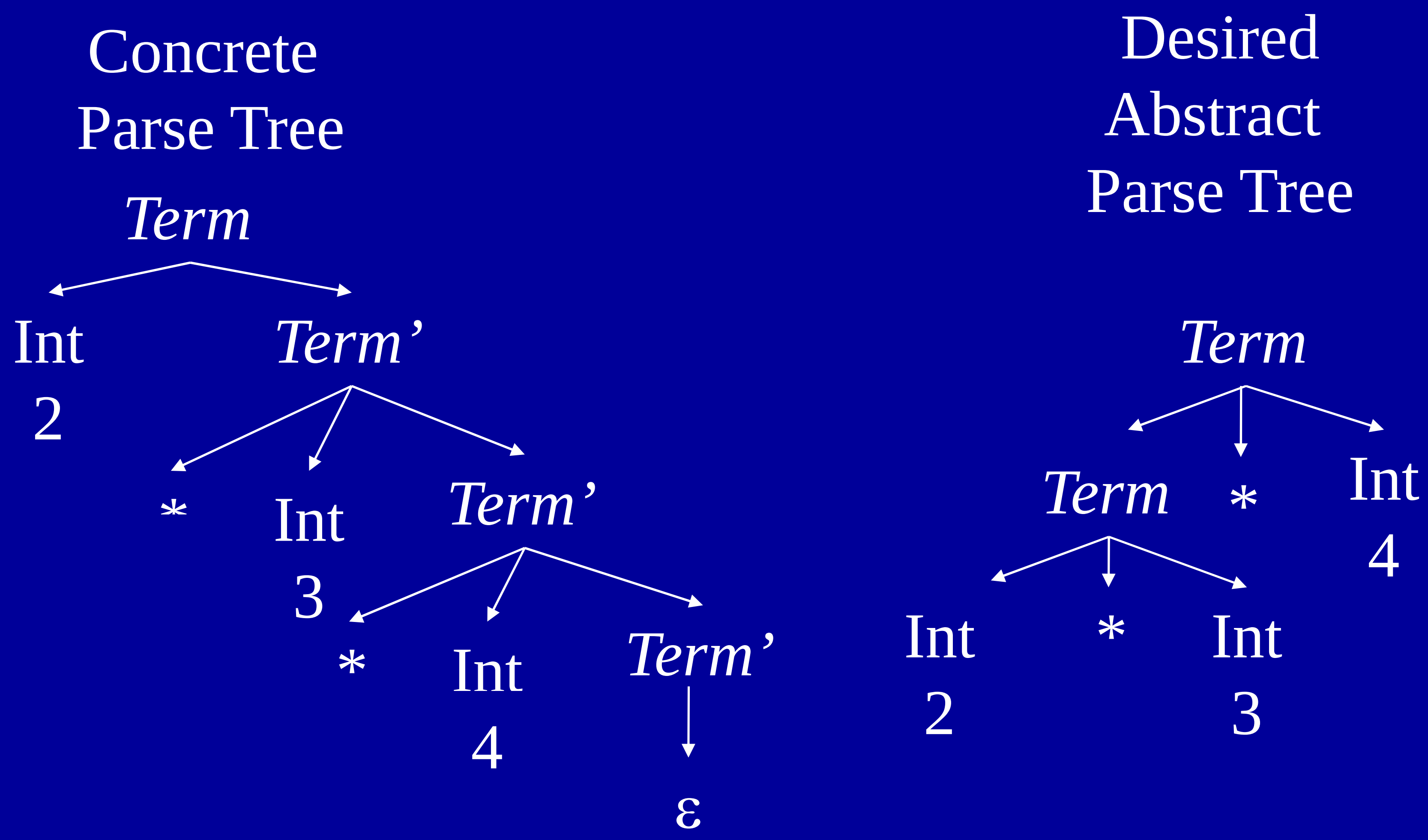
Building A Parse Tree

- Have each procedure return the section of the parse tree for the part of the string it parsed
- Use exceptions to make code structure clean

Building Parse Tree In Example

```
Term()
    if (token = Int n)
            oldToken = token; token = NextToken();
            node = TermPrime();
            if (node == NULL) return oldToken;
            else return(new TermNode(oldToken, node);
    else throw SyntaxError
TermPrime()
    if (token = *) || (token = /)
            first = token; next = NextToken();
            if (next = Int n)
                        token = NextToken();
                        return(new TermPrimeNode(first, next, TermPrime())
            else throw SyntaxError
    else return(NULL)
```

Parse Tree for 2*3*4



Why Use Hand-Coded Parser?

- Why not use parser generator?
- What do you do if your parser doesn't work?
  - Recursive descent parser – write more code
  - Parser generator
    - Hack grammar
    - But if parser generator doesn't work, nothing you can do
- If you have complicated grammar
  - Increase chance of going outside comfort zone of parser generator
  - Your parser may NEVER work

Bottom Line

- Recursive descent parser properties
  - Probably more work
  - But less risk of a disaster - you can almost always make a recursive descent parser work
  - May have easier time dealing with resulting code
    - Single language system
    - No need to deal with potentially flaky parser generator
    - No integration issues with automatically generated code
- If your parser development time is small compared to rest of project, or you have a really complicated language, use hand-coded recursive descent parser

Summary

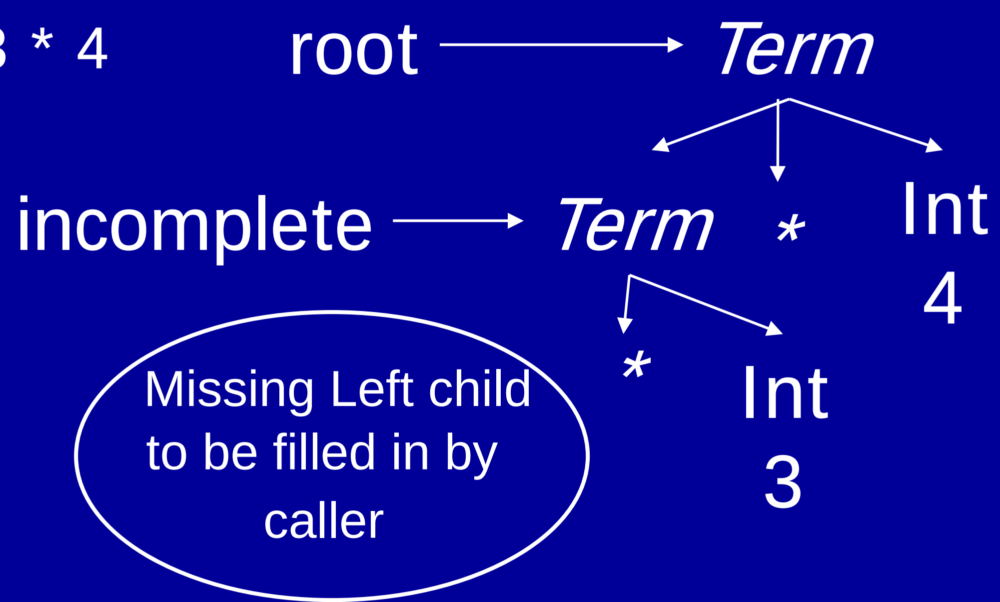
- Top-Down Parsing
- Use Lookahead to Avoid Backtracking
- Parser is

  Hand-Coded

  - Set of Mutually Recursive Procedures

Direct Generation of Abstract Tree

- TermPrime builds an incomplete tree
  - Missing leftmost child
  - Returns root and incomplete node
- (root, incomplete) = TermPrime()
  - Called with token = *
  - Remaining tokens = 3 * 4



Code for Term

Input to parse

2*3*4

```
Term()
   if (token = Int n)
         leftmostInt = token; token = NextToken();
         (root, incomplete) = TermPrime();
         if (root == NULL) return leftmostInt;
         incomplete.leftChild    leftmostInt;
         return root;
   else throw SyntaxError
```

token → Int 2

Code for Term

Input to parse

2*3*4

```
Term()
    if (token = Int n)
        leftmostInt = token; token = NextToken();
        (root, incomplete) = TermPrime();
        if (root == NULL) return leftmostInt;
        incomplete.leftChild    leftmostInt;
        return root;
    else throw SyntaxError
```

token → Int
2

Code for Term

Input to parse

2*3*4

```
Term()
    if (token = Int n)
        leftmostInt = token; token = NextToken();
        (root, incomplete) = TermPrime();
        if (root == NULL) return leftmostInt;
        incomplete.leftChild    leftmostInt;
        return root;
    else throw SyntaxError
```

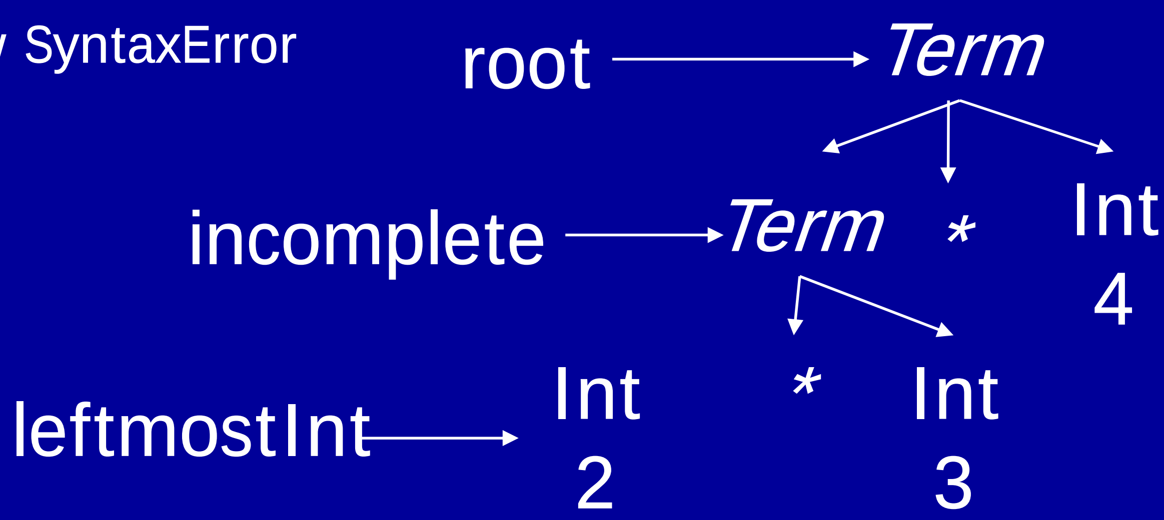
token $\longrightarrow$ Int 2

Code for Term

Input to parse

2*3*4

```
Term()
    if (token = Int n)
        leftmostInt = token; token = NextToken();
        (root, incomplete) = TermPrime();
        if (root == NULL) return leftmostInt;
        incomplete.leftChild    leftmostInt;
        return root;
    else throw SyntaxError
```

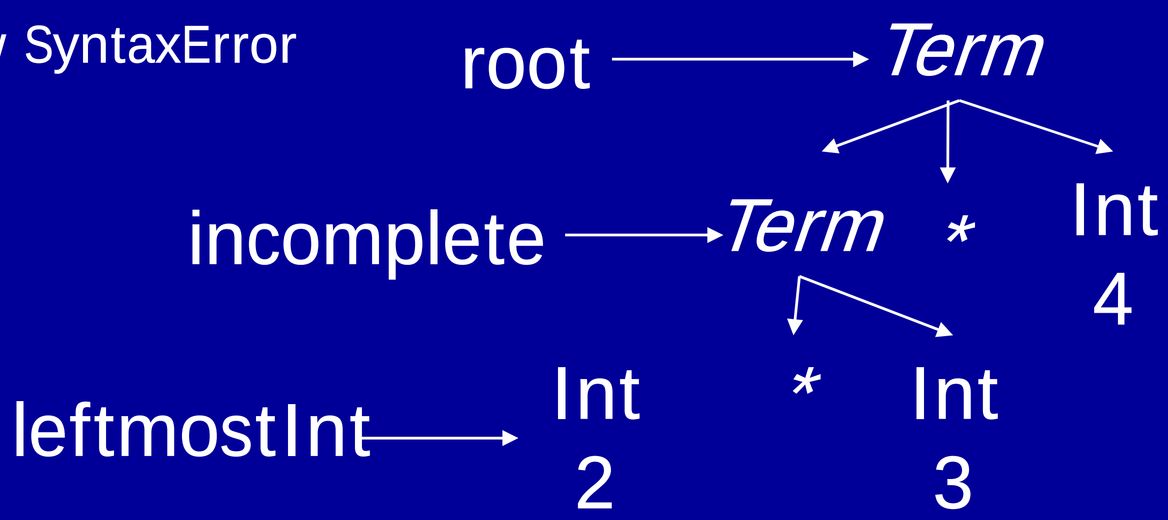


Code for Term

Input to parse

2*3*4

```
Term()
   if (token = Int n)
        leftmostInt = token; token = NextToken();
        (root, incomplete) = TermPrime();
        if (root == NULL) return leftmostInt;
        incomplete.leftChild    leftmostInt;
        return root;
   else throw SyntaxError
```

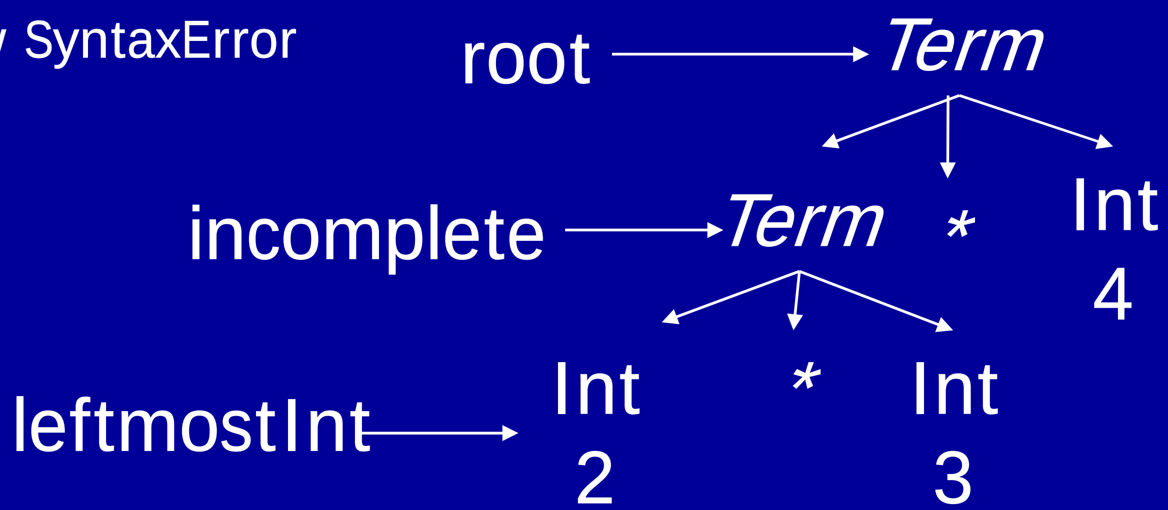


Code for Term

```
Term()
    if (token = Int n)
        leftmostInt = token; token = NextToken();
        (root, incomplete) = TermPrime();
        if (root == NULL) return leftmostInt;
        incomplete.leftChild    leftmostInt;
        return root;
    else throw SyntaxError
```

Input to parse

2*3*4



Code for TermPrime

Missing left child to be filled in by caller

```
TermPrime()
    if (token = *) || (token = /)
        op = token; next = NextToken();
        if (next = Int n)
            token = NextToken();
            (root, incomplete) = TermPrime();
            if (root == NULL)
                root = new ExprNode(NULL, op, next);
                return (root, root);
            else    newChild = new ExprNode(NULL, op, next);
                incomplete.leftChild = newChild;
                return(root, newChild);
        else throw SyntaxError
    else return(NULL,NULL)
```

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