MIT 6.035
Top-Down Parsing

Martin Rinard
Laboratory for Computer Science
Massachusetts Institute of Technology

Orientation

- Language specification
  - Lexical structure – regular expressions
  - Syntactic structure – grammar
- This Lecture - recursive descent parsers
  - Code parser as set of mutually recursive procedures
  - Structure of program matches structure of grammar

Starting Point

- Assume lexical analysis has produced a sequence of tokens
  - Each token has a type and value
  - Types correspond to terminals
  - Values to contents of token read in
- Examples
  - Int 549 – integer token with value 549 read in
  - if - if keyword, no need for a value
  - AddOp + - add operator, value +

Basic Approach

- Start with Start symbol
- Build a leftmost derivation
  - If leftmost symbol is nonterminal, choose a production and apply it
  - If leftmost symbol is terminal, match against input
  - If all terminals match, have found a parse!
  - Key: find correct productions for nonterminals

Graphical Illustration of Leftmost Derivation

Sentential Form

$NT_1 \; T_1 \; T_2 \; T_3 \; NT_2 \; NT_3$

Apply Production Here

Not Here

Grammar for Parsing Example

$Start \rightarrow Expr$

$Expr \rightarrow Expr + Term$

$Expr \rightarrow Expr - Term$

$Expr \rightarrow Term$

$Term \rightarrow Term *$ Int

$Term \rightarrow Term /$ Int

$Term \rightarrow$ Int

- Set of tokens is
  { +, -, *, /, Int }, where
  Int = [0-9][0-9]*
- For convenience, may represent each Int n token by n

Parsing Example

Parse Tree

Start

Current Position in Parse Tree

Remaining Input

2-2*2

Sentential Form

Start

Parsing Example

Parse Tree

Start

Expr

Current Position in Parse Tree

Remaining Input

2-2*2

Sentential Form

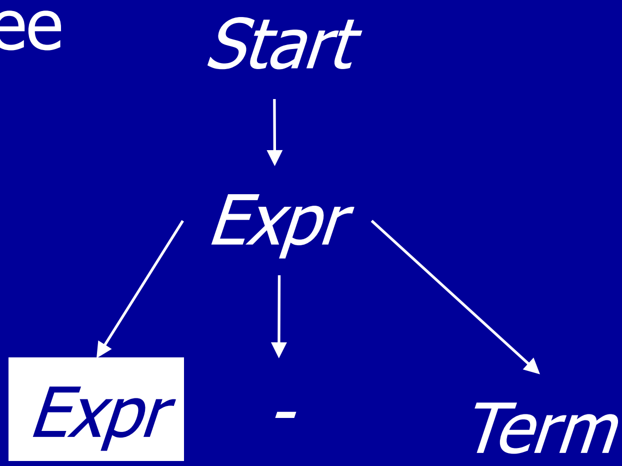
Expr

Applied Production

Start $\rightarrow$ *Expr*

Parsing Example

Parse Tree



Remaining Input

2-2*2

Sentential Form

Expr - Term

$Expr \rightarrow Expr + Term$

$\boxed{Expr \rightarrow Expr - Term}$

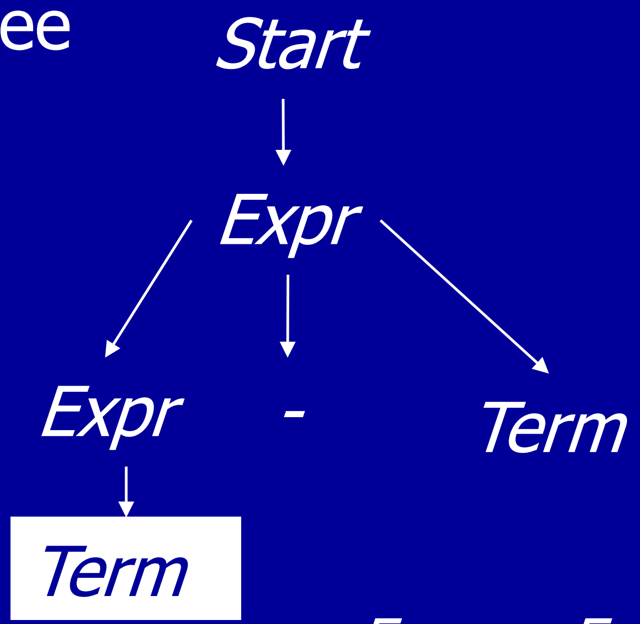
$Expr \rightarrow Term$

Applied Production

$Expr \rightarrow Expr - Term$

Parsing Example

Parse Tree



Expr $\rightarrow$ *Expr + Term*

Expr $\rightarrow$ *Expr - Term*

Expr $\rightarrow$ *Term*

Remaining Input

2-2*2

Sentential Form

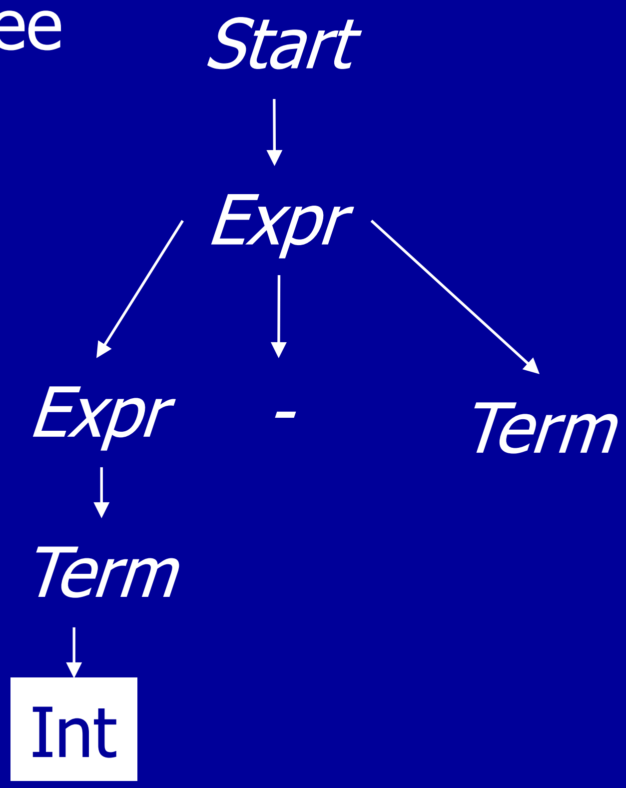
Term - Term

Applied Production

Expr $\rightarrow$ *Term*

Parsing Example

Parse Tree



Remaining Input

2-2*2

Sentential Form

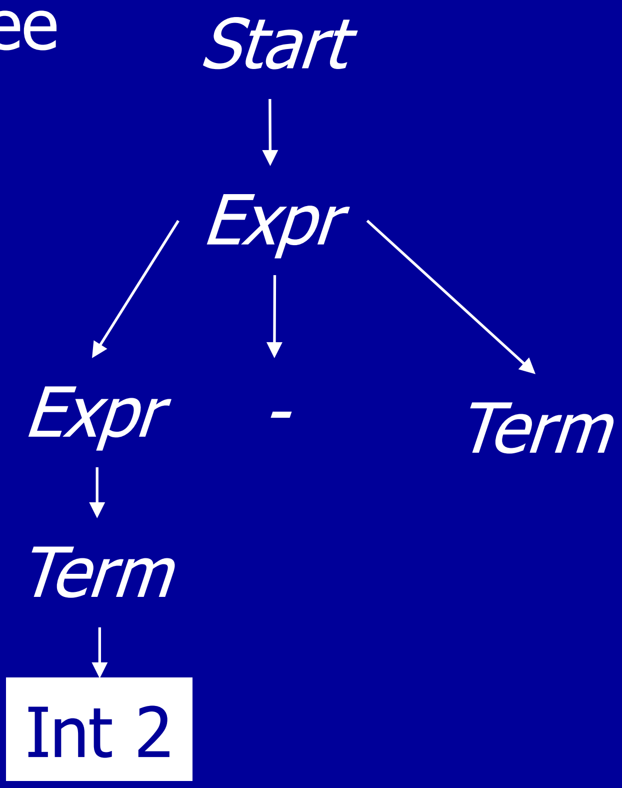
Int - *Term*

Applied Production

Term $\rightarrow$ Int

Parsing Example

Parse Tree



Match Input Token!

Remaining Input

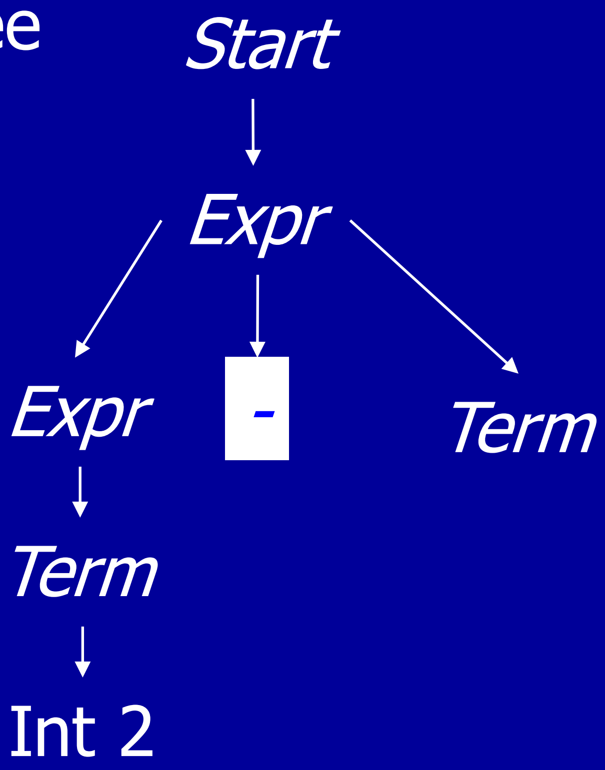
2-2*2

Sentential Form

2 - *Term*

Parsing Example

Parse Tree



Match Input Token!

Remaining Input

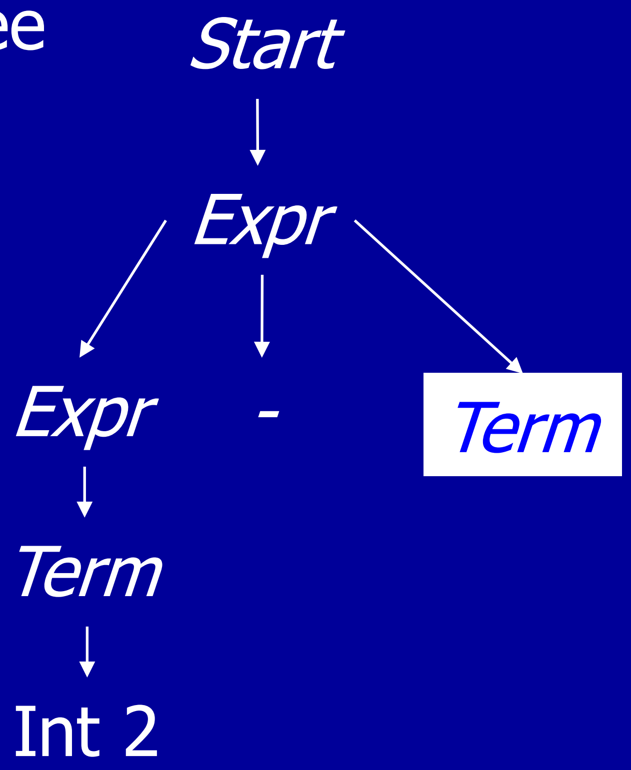
-2*2

Sentential Form

2 - *Term*

Parsing Example

Parse Tree



Match Input Token!

Remaining Input

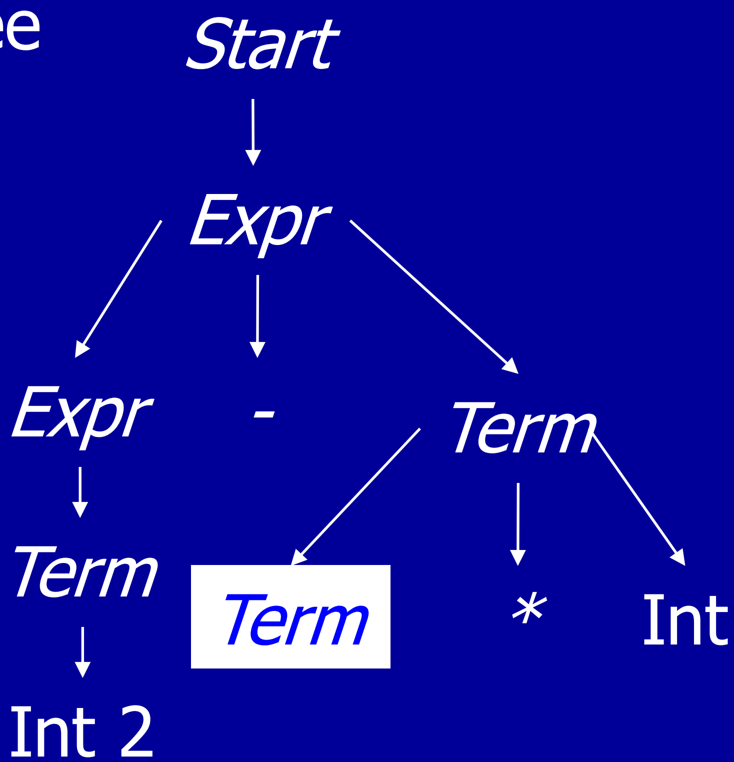
2*2

Sentential Form

2 - *Term*

Parsing Example

Parse Tree



Remaining Input

2*2

Sentential Form

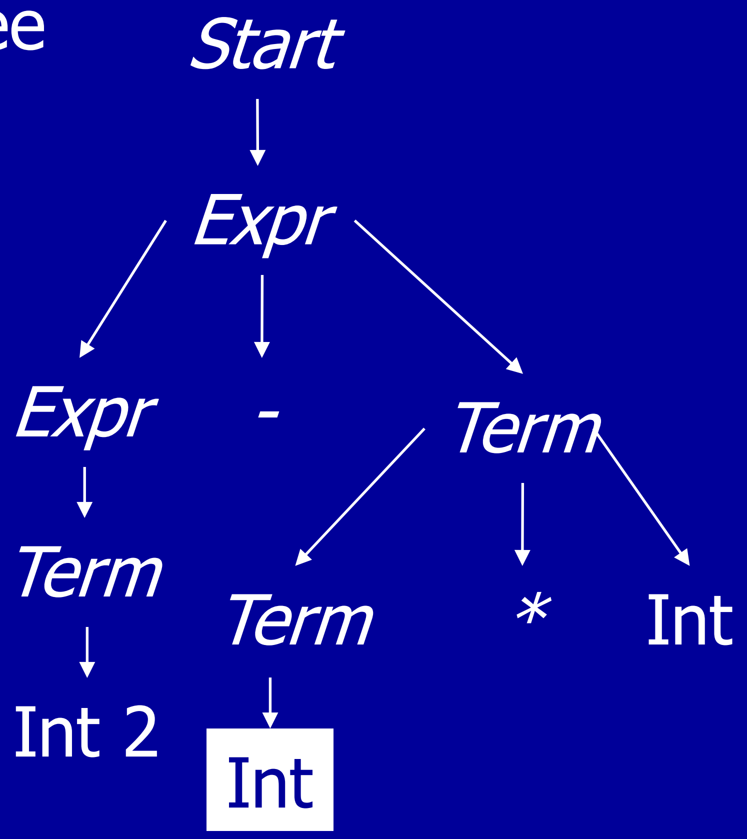
2 - *Term**Int

Applied Production

$Term \rightarrow Term * \text{Int}$

Parsing Example

Parse Tree



Remaining Input

2*2

Sentential Form

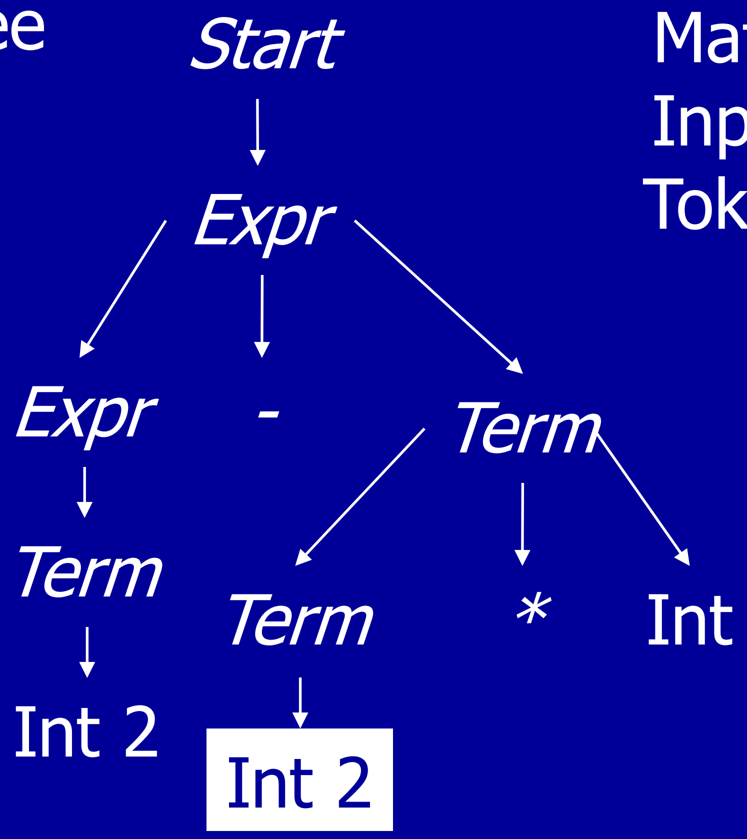
2 - Int * Int

Applied Production

Term → Int

Parsing Example

Parse
Tree



Match
Input
Token!

Remaining Input

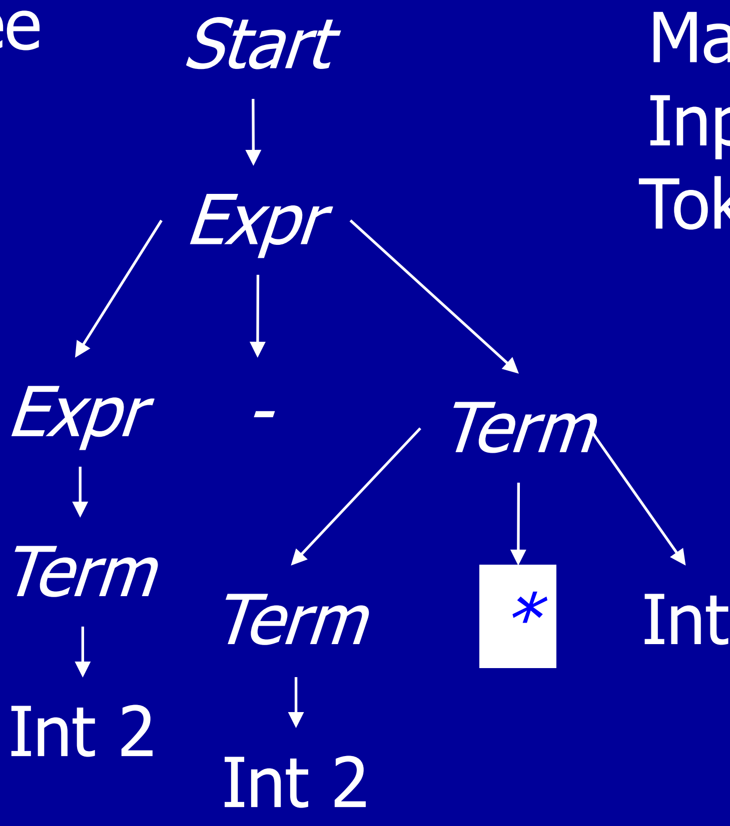
2*2

Sentential Form

2 - 2 * Int

Parsing Example

Parse Tree



Match Input Token!

Remaining Input

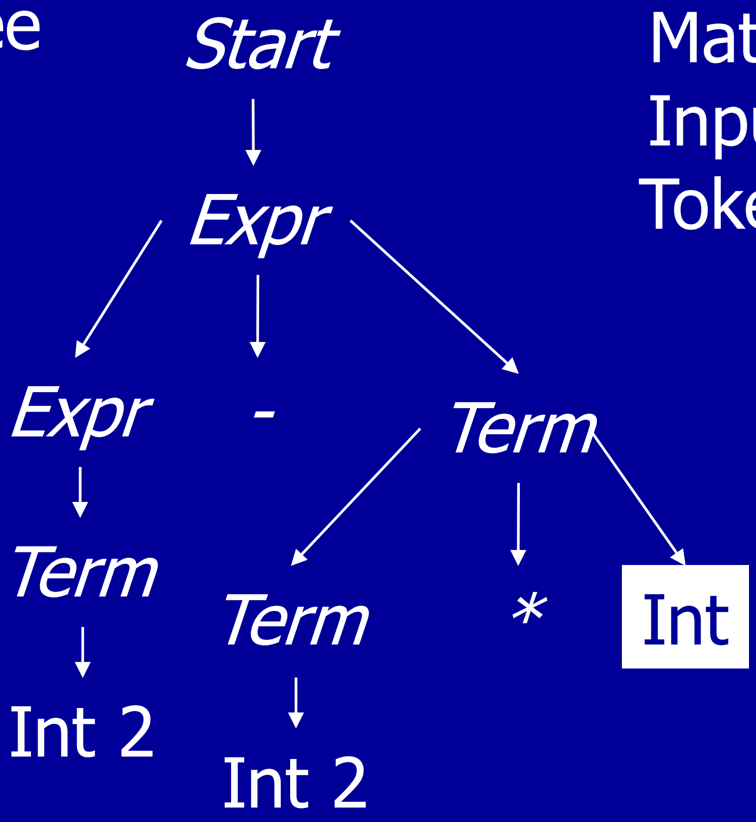
*2

Sentential Form

2 - 2 * Int

Parsing Example

Parse Tree



Match Input Token!

Remaining Input

2

Sentential Form

2 - 2 * Int

Parsing Example

Parse Tree

Start

Expr

Expr - *Term*

Term *Term* * Int 2

Int 2 Int 2

Parse Complete!

Remaining Input

2

Sentential Form

2 - 2*2

Summary

- Three Actions (Mechanisms)
  - Apply production to expand current nonterminal in parse tree
  - Match current terminal (consuming input)
  - Accept the parse as correct
- Parser generates preorder traversal of parse tree
  - visit parents before children
  - visit siblings from left to right

Policy Problem

- Which production to use for each nonterminal?
- Classical Separation of Policy and Mechanism
- One Approach: Backtracking
  - Treat it as a search problem
  - At each choice point, try next alternative
  - If it is clear that current try fails, go back to previous choice and try something different
- General technique for searching
- Used a lot in classical AI and natural language processing (parsing, speech recognition)

Backtracking Example

Parse Tree *Start*

Remaining Input

2-2*2

Sentential Form

Start

Backtracking Example

Parse Tree

$$\mathit{Start} \rightarrow \boxed{\mathit{Expr}}$$

Remaining Input

2-2*2

Sentential Form

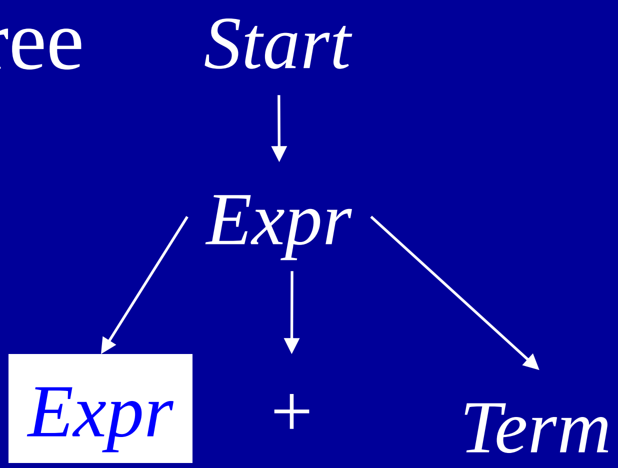
Expr

Applied Production

$\mathit{Start} \rightarrow \mathit{Expr}$

Backtracking Example

Parse Tree



Remaining Input

2-2*2

Sentential Form

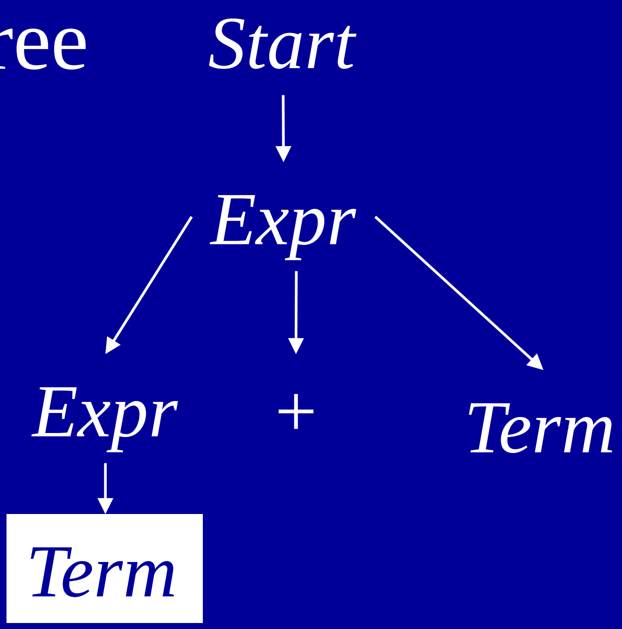
Expr + *Term*

Applied Production

$Expr \rightarrow Expr + Term$

Backtracking Example

Parse Tree



Remaining Input

2-2*2

Sentential Form

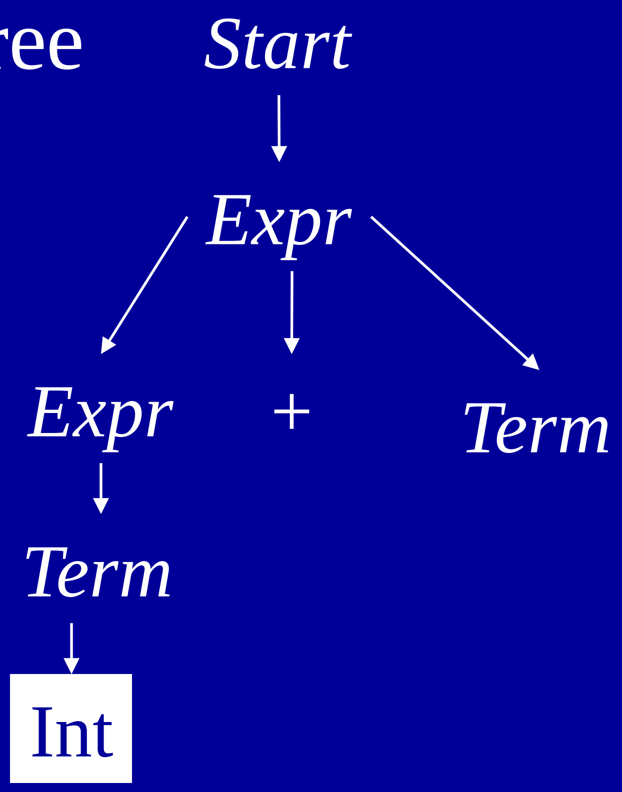
Term + *Term*

Applied Production

$Expr \rightarrow Term$

Backtracking Example

Parse Tree



Match Input Token!

Remaining Input

2-2*2

Sentential Form

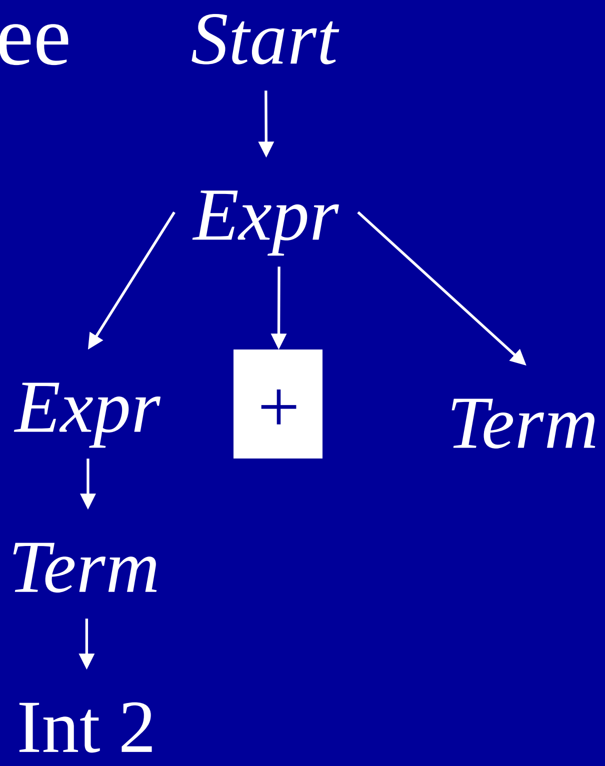
Int + *Term*

Applied Production

Term → Int

Backtracking Example

Parse Tree



Can't Match Input Token!

Remaining Input

-2*2

Sentential Form

2 - *Term*

Applied Production

Term → Int

Backtracking Example

Parse Tree

Start → $\boxed{Expr}$

So Backtrack!

Remaining Input

2-2*2

Sentential Form

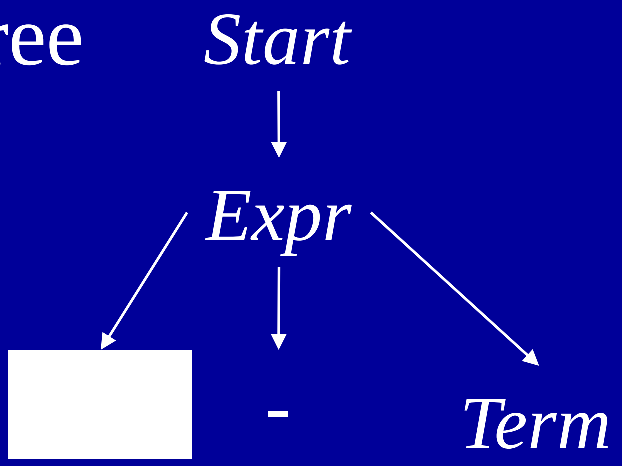
Expr

Applied Production

Start → *Expr*

Backtracking Example

Parse Tree



Remaining Input

2-2*2

Sentential Form

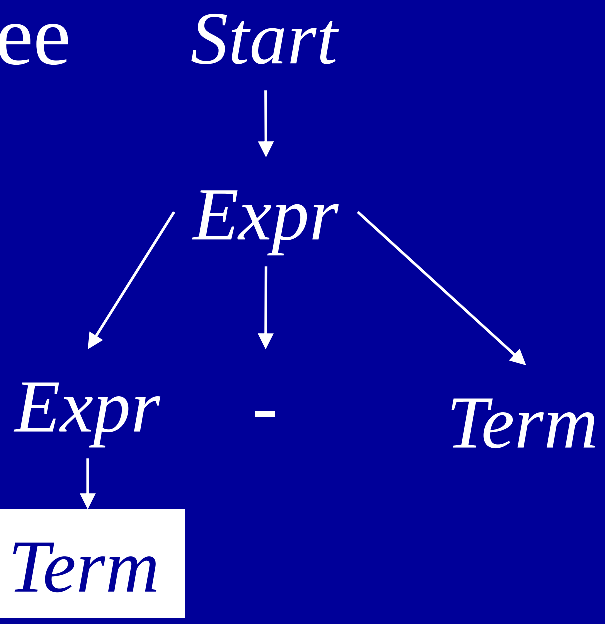
Expr - Term

Applied Production

$Expr \rightarrow Expr - Term$

Backtracking Example

Parse Tree



Remaining Input

2-2*2

Sentential Form

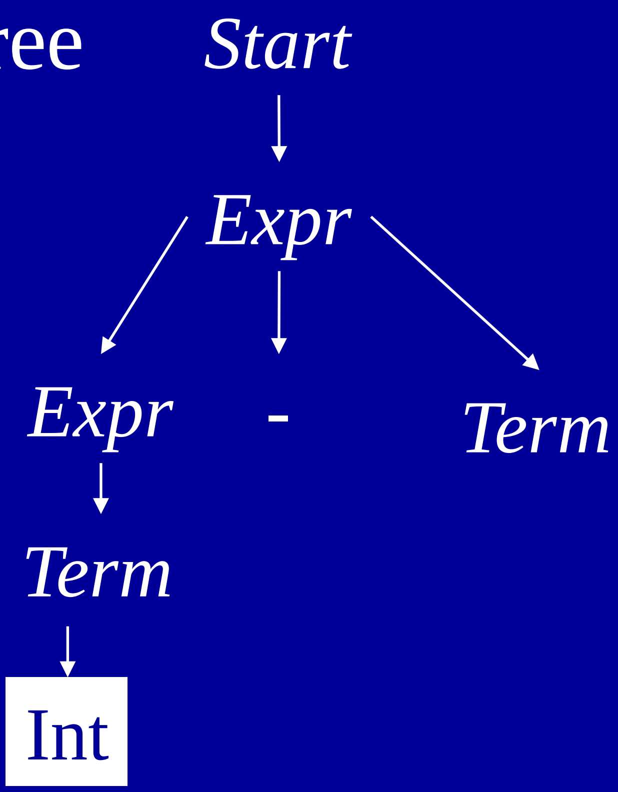
Term - *Term*

Applied Production

$Expr \rightarrow Term$

Backtracking Example

Parse Tree



Remaining Input

2-2*2

Sentential Form

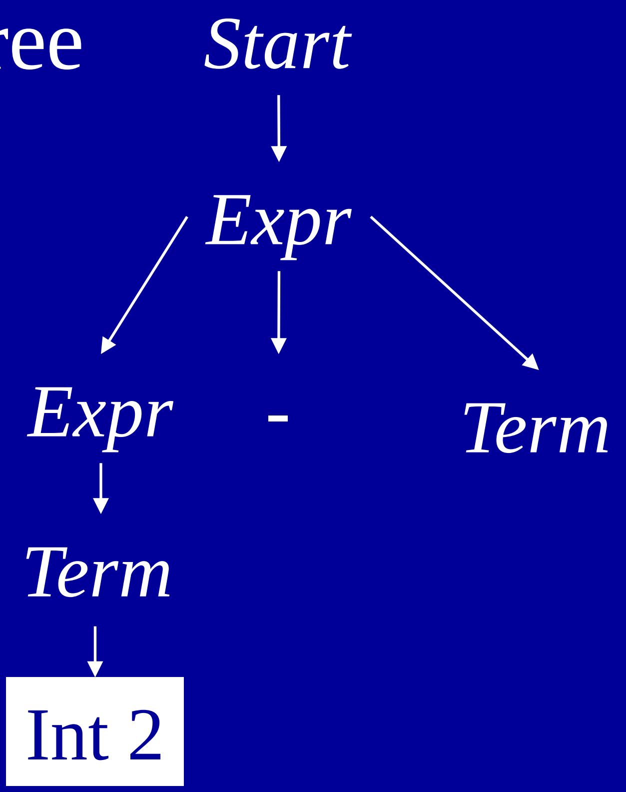
Int - *Term*

Applied Production

Term $\rightarrow$ Int

Backtracking Example

Parse Tree



Match Input Token!

Remaining Input

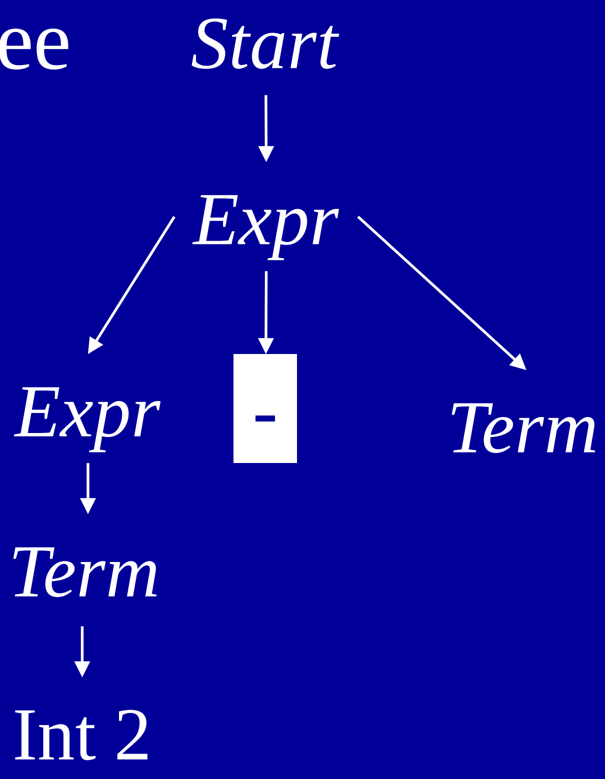
-2*2

Sentential Form

2 - *Term*

Backtracking Example

Parse
Tree



Match
Input
Token!

Remaining Input

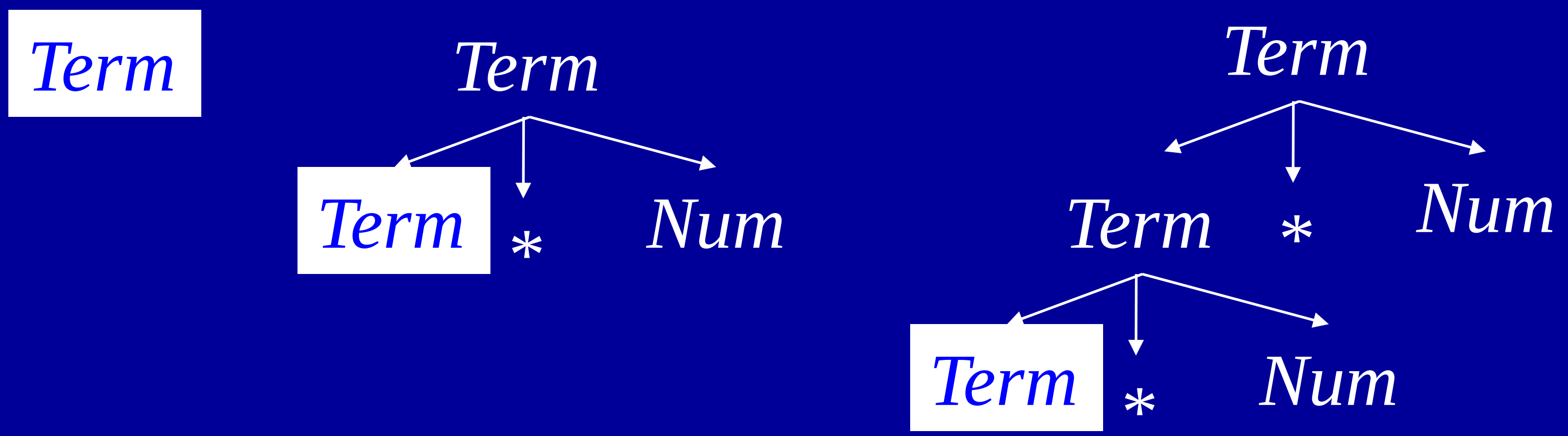
2*2

Sentential Form

2 - *Term*

Left Recursion + Top-Down Parsing = Infinite Loop

- Example Production: $Term \rightarrow Term * Num$
- Potential parsing steps:

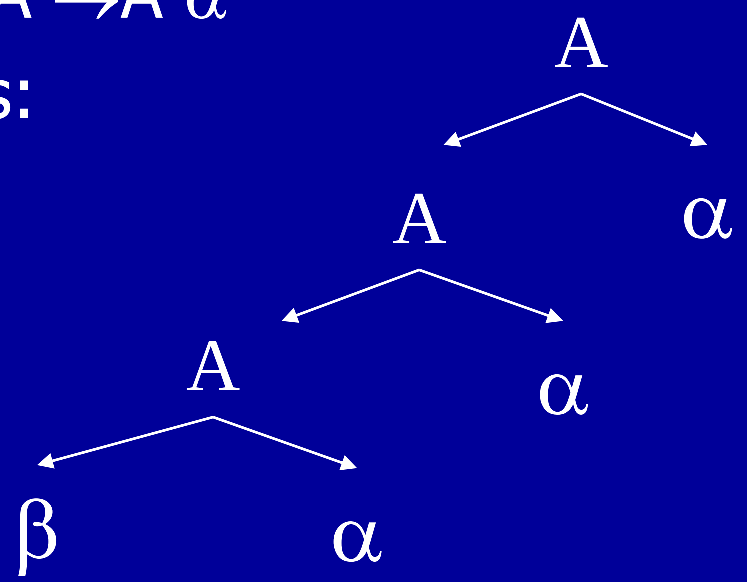


General Search Issues

- Three components
  - Search space (parse trees)
  - Search algorithm (parsing algorithm)
  - Goal to find (parse tree for input program)
- Would like to (but can't always) ensure that
  - Find goal (hopefully quickly) if it exists
  - Search terminates if it does not
- Handled in various ways in various contexts
  - Finite search space makes it easy
  - Exploration strategies for infinite search space
  - Sometimes one goal more important (model checking)
- For parsing, hack grammar to remove left recursion

Eliminating Left Recursion

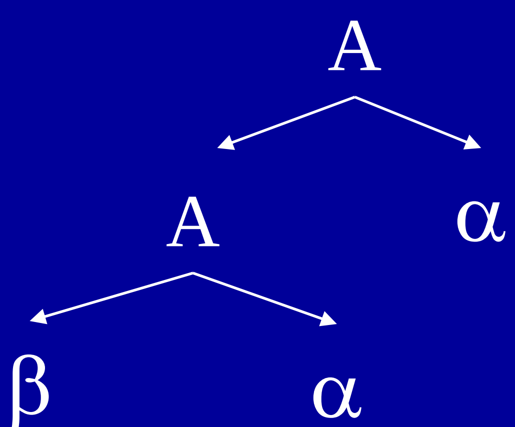
- Start with productions of form
  - $A \to A\ \alpha$
  - $A \to \beta$
  - α, β sequences of terminals and nonterminals that do not start with A
- Repeated application of $A \to A\ \alpha$ builds parse tree like this:



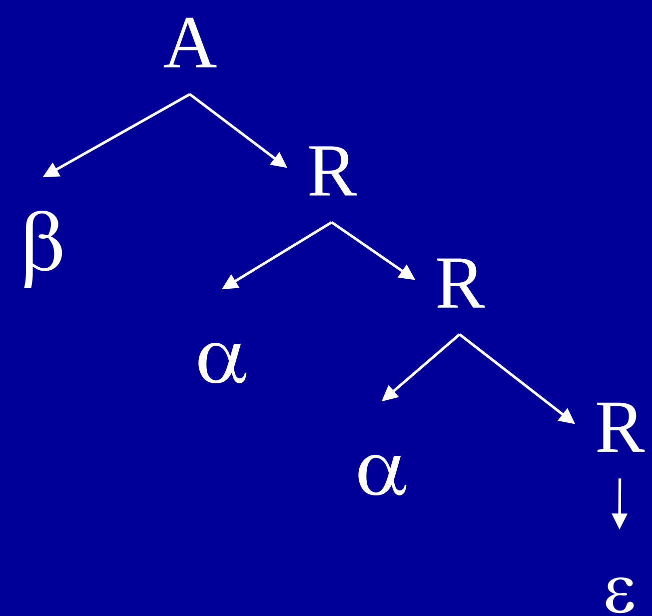
Eliminating Left Recursion

- Replacement productions
  - $A \rightarrow A\ \alpha$ $\quad$ $A \rightarrow \beta\ R$ $\quad$ R is a new nonterminal
  - $A \rightarrow \beta$ $\quad$ $R \rightarrow \alpha\ R$
  - $\quad$ $R \rightarrow \varepsilon$

Old Parse Tree



New Parse Tree



Hacked Grammar

Original Grammar Fragment

$Term \rightarrow Term * \text{Int}$

$Term \rightarrow Term \, / \, \text{Int}$

$Term \rightarrow \text{Int}$

New Grammar Fragment

$Term \rightarrow \text{Int} \; Term'$

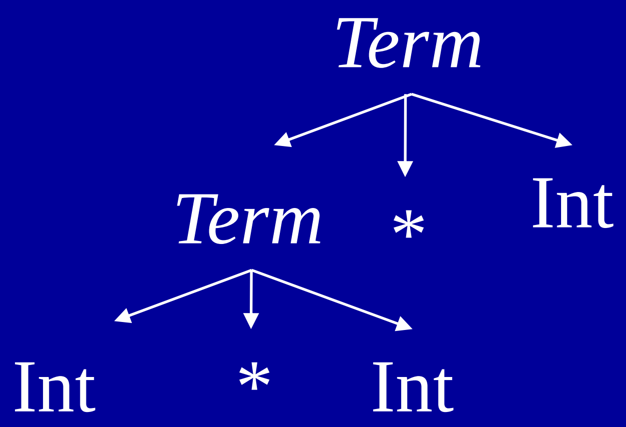
$Term' \rightarrow * \, \text{Int} \; Term'$

$Term' \rightarrow / \, \text{Int} \; Term'$

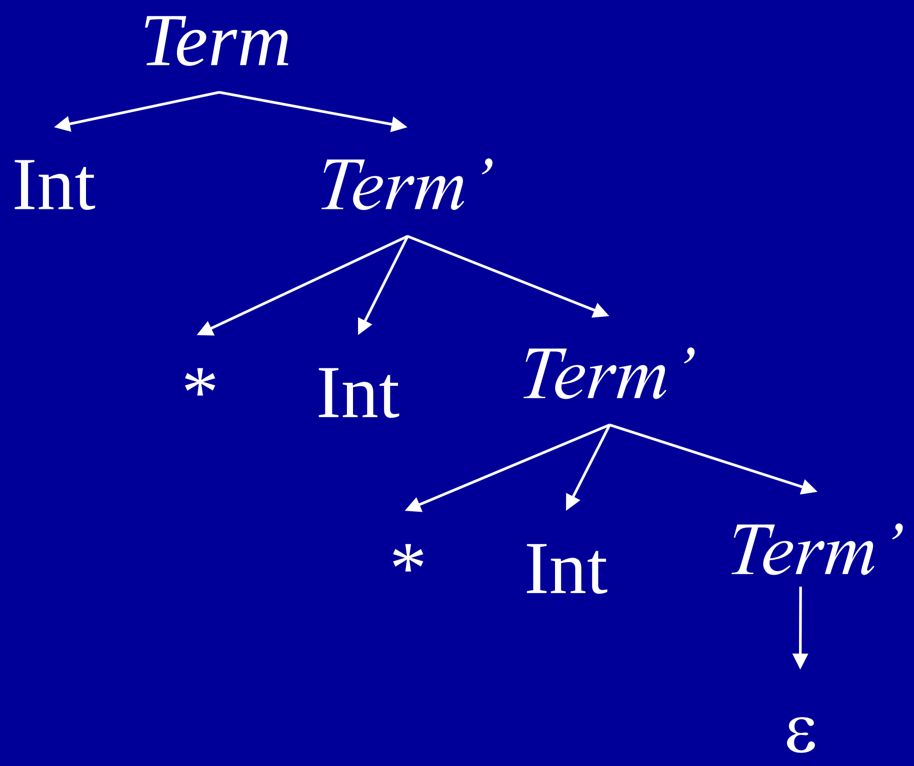
$Term' \rightarrow \varepsilon$

Parse Tree Comparisons

Original Grammar



New Grammar



Eliminating Left Recursion

- Changes search space exploration algorithm
  - Eliminates direct infinite recursion
  - But grammar less intuitive
- Sets things up for predictive parsing

Predictive Parsing

- Alternative to backtracking
- Useful for programming languages, which can be designed to make parsing easier
- Basic idea
  - Look ahead in input stream
  - Decide which production to apply based on next tokens in input stream
  - We will use one token of lookahead

Predictive Parsing Example Grammar

$Start \rightarrow Expr$

$Expr \rightarrow Term\ Expr'$

$Expr' \rightarrow +\ Term\ Expr'$

$Expr' \rightarrow -\ Term\ Expr'$

$Expr' \rightarrow \varepsilon$

$Term \rightarrow \mathrm{Int}\ Term'$

$Term' \rightarrow *\ \mathrm{Int}\ Term'$

$Term' \rightarrow /\ \mathrm{Int}\ Term'$

$Term' \rightarrow \varepsilon$

Choice Points

- Assume *Term′* is current position in parse tree
- Have three possible productions to apply

  Term′ $\rightarrow$ *** Int *Term′*

  Term′ $\rightarrow$ */* Int *Term′*

  Term′ $\rightarrow$ ε
- Use next token to decide
  - If next token is *, apply *Term′* $\rightarrow$ *** Int *Term′*
  - If next token is /, apply *Term′* $\rightarrow$ */* Int *Term′*
  - Otherwise, apply *Term′* $\rightarrow$ ε

Predictive Parsing + Hand Coding = Recursive Descent Parser

- One procedure per nonterminal *NT*
  - Productions $NT \rightarrow \beta_1$, …, $NT \rightarrow \beta_n$
  - Procedure examines the current input symbol T to determine which production to apply
    - If $T \in \mathrm{First}(\beta_k)$
    - Apply production k
    - Consume terminals in β_k (check for correct terminal)
    - Recursively call procedures for nonterminals in β_k
  - Current input symbol stored in global variable token
- Procedures return
  - true if parse succeeds
  - false if parse fails

Example

```
Boolean Term()
    if (token = Int n) token = NextToken(); return(TermPrime())
    else return(false)
Boolean TermPrime()
    if (token = *)
        token = NextToken();
        if (token = Int n) token = NextToken(); return(TermPrime())
        else return(false)
    else if (token = /)
        token = NextToken();
        if (token = Int n) token = NextToken(); return(TermPrime())
        else return(false)
    else return(true)
```

$$Term \rightarrow \text{Int } Term'$$
$$Term' \rightarrow *\,\text{Int } Term'$$
$$Term' \rightarrow /\,\text{Int } Term'$$
$$Term' \rightarrow \varepsilon$$

Multiple Productions With Same Prefix in RHS

- Example Grammar

  NT $\rightarrow$ if then

  NT $\rightarrow$ if then else
- Assume *NT* is current position in parse tree, and if is the next token
- Unclear which production to apply
  - Multiple k such that T $\in$ First(β_k)
  - if $\in$ First(if then)
  - if $\in$ First(if then else)

Solution: Left Factor the Grammar

- New Grammar Factors Common Prefix Into Single Production

  $NT \rightarrow$ if then NT'

  $NT' \rightarrow$ else

  $NT' \rightarrow \varepsilon$
- No choice when next token is if!
- All choices have been unified in one production.

Nonterminals

- What about productions with nonterminals?

$$NT \rightarrow NT_1\ \alpha_1$$

$$NT \rightarrow NT_2\ \alpha_2$$

- Must choose based on possible first terminals that NT_1 and NT_2 can generate
- What if NT_1 or NT_2 can generate ε?
  - Must choose based on α_1 and α_2

NT derives ε

- Two rules
  - $NT \rightarrow \varepsilon$ implies *NT* derives ε
  - $NT \rightarrow NT_1 \ldots NT_n$ and for all $1 \leq i \leq n$ NT_i derives ε implies *NT* derives ε

Fixed Point Algorithm for Derives ε

```
for all nonterminals NT
   set NT derives ε to be false
for all productions of the form NT → ε
    set NT derives ε to be true
while (some NT derives ε changed in last iteration)
    for all productions of the form NT → NT_1 ... NT_n
        if (for all 1≤i ≤n NT_i derives ε)
                set NT derives ε to be true
```

First(β)

- T $\in$ First(β) if T can appear as the first symbol in a derivation starting from β
  1) $T \in$ First(T)
  2) First(S) $\subseteq$ First($S\,\beta$)
  3) NT derives ε implies First(β) $\subseteq$ First($NT\,\beta$)
  4) $NT \rightarrow S\,\beta$ implies First(S β) $\subseteq$ First(NT)

- Notation
  - T is a terminal, NT is a nonterminal, S is a terminal or nonterminal, and β is a sequence of terminals or nonterminals

Rules + Request Generate System of Subset Inclusion Constraints

Grammar

$Term' \rightarrow$ * Int $Term'$

$Term' \rightarrow$ / Int $Term'$

$Term' \rightarrow \varepsilon$

Request: What is First($Term'$)?

Constraints

First(* Num $Term'$) $\subseteq$ First($Term'$)

First(/ Num $Term'$) $\subseteq$ First($Term'$)

First(*) $\subseteq$ First(* Num $Term'$)

First(/) $\subseteq$ First(/ Num $Term'$)

* $\in$ First(*)

/ $\in$ First(/)

Rules

1) $T \in$ First(T)
2) First(S) $\subseteq$ First($S\ \beta$)
3) NT derives ε implies First(β) $\subseteq$ First($NT\ \beta$)
4) $NT \rightarrow S\ \beta$ implies First(S β) $\subseteq$ First(NT)

Constraint Propagation Algorithm

Constraints

First($* \; Num \; Term'$) $\subseteq$ First($Term'$)

First($/ \; Num \; Term'$) $\subseteq$ First($Term'$)

First(*) $\subseteq$ First($* \; Num \; Term'$)

First(/) $\subseteq$ First($/ \; Num \; Term'$)

$* \in$ First(*)

$/ \in$ First(/)

Solution

First($Term'$) = {}

First($* \; Num \; Term'$) = {}

First($/ \; Num \; Term'$) = {}

First(*) = {*}

First(/) = {/}

Initialize Sets to {}

Propagate Constraints Until Fixed Point

Constraint Propagation Algorithm

Constraints

First($*$ *Num Term′*) $\subseteq$ First(*Term′*)

First($/$ *Num Term′*) $\subseteq$ First(*Term′*)

First($*$) $\subseteq$ First($*$ *Num Term′*)

First($/$) $\subseteq$ First($/$ *Num Term′*)

$* \in$ First($*$)

$/ \in$ First($/$)

Solution

First(*Term′*) = {}

First($*$ *Num Term′*) = {}

First($/$ *Num Term′*) = {}

First($*$) = {$*$}

First($/$) = {$/$}

Grammar

$Term' \rightarrow * \text{ Int } Term'$

$Term' \rightarrow / \text{ Int } Term'$

$Term' \rightarrow \varepsilon$

Constraint Propagation Algorithm

Constraints

$\text{First}(*\ Num\ Term') \subseteq \text{First}(Term')$

$\text{First}(/\ Num\ Term') \subseteq \text{First}(Term')$

$\text{First}(*) \subseteq \text{First}(*\ Num\ Term')$

$\text{First}(/) \subseteq \text{First}(/\ Num\ Term')$

$* \in \text{First}(*)$

$/ \in \text{First}(/)$

Solution

$\text{First}(Term') = \{\}$

$\text{First}(*\ Num\ Term') = \{*\}$

$\text{First}(/\ Num\ Term') = \{/\}$

$\text{First}(*) = \{*\}$

$\text{First}(/) = \{/\}$

Grammar

$Term' \rightarrow *\ \text{Int}\ Term'$

$Term' \rightarrow /\ \text{Int}\ Term'$

$Term' \rightarrow \varepsilon$

Constraint Propagation Algorithm

Constraints

$\text{First}(*\ Num\ Term') \subseteq \text{First}(Term')$

$\text{First}(/\ Num\ Term') \subseteq \text{First}(Term')$

$\text{First}(*) \subseteq \text{First}(*\ Num\ Term')$

$\text{First}(/) \subseteq \text{First}(/\ Num\ Term')$

$* \in \text{First}(*)$

$/ \in \text{First}(/)$

Solution

$\text{First}(Term') = \{*,/\}$

$\text{First}(*\ Num\ Term') = \{*\}$

$\text{First}(/\ Num\ Term') = \{/\}$

$\text{First}(*) = \{*\}$

$\text{First}(/) = \{/\}$

Grammar

$Term' \rightarrow *\ \text{Int}\ Term'$

$Term' \rightarrow /\ \text{Int}\ Term'$

$Term' \rightarrow \varepsilon$

Constraint Propagation Algorithm

Constraints

First($*$ *Num Term′*) $\subseteq$ First(*Term′*)

First($/$ *Num Term′*) $\subseteq$ First(*Term′*)

First($*$) $\subseteq$ First($*$ *Num Term′*)

First($/$) $\subseteq$ First($/$ *Num Term′*)

$* \in$ First($*$)

$/ \in$ First($/$)

Solution

First(*Term′*) = {$*$,$/$}

First($*$ *Num Term′*) = {$*$}

First($/$ *Num Term′*) = {$/$}

First($*$) = {$*$}

First($/$) = {$/$}

Grammar

Term′ $\rightarrow$ $*$ Int *Term′*

Term′ $\rightarrow$ $/$ Int *Term′*

Term′ $\rightarrow$ ε

Building A Parse Tree

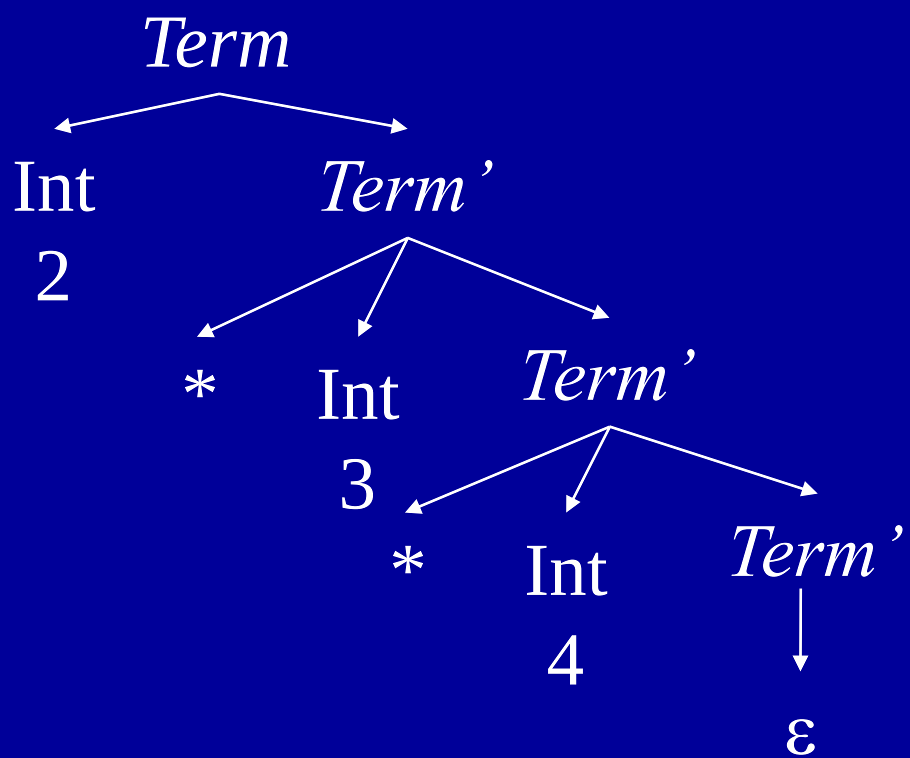
- Have each procedure return the section of the parse tree for the part of the string it parsed
- Use exceptions to make code structure clean

Building Parse Tree In Example

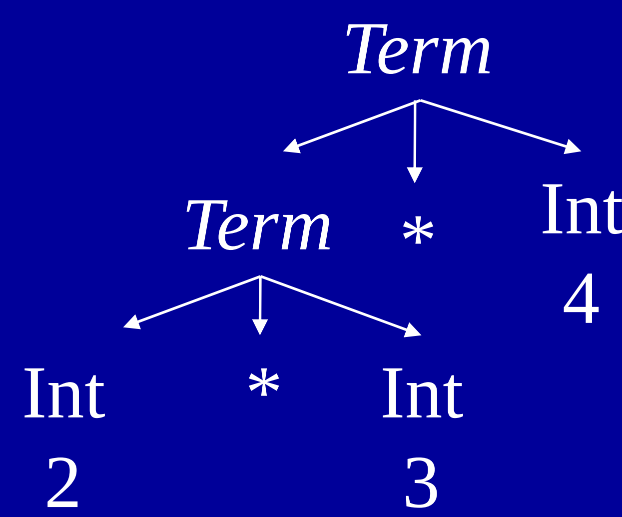
```
Term()
    if (token = Int n)
        oldToken = token; token = NextToken();
        node = TermPrime();
        if (node == NULL) return oldToken;
        else return(new TermNode(oldToken, node);
    else throw SyntaxError
TermPrime()
    if (token = *) || (token = /)
        first = token; next = NextToken();
        if (next = Int n)
            token = NextToken();
            return(new TermPrimeNode(first, next, TermPrime())
        else throw SyntaxError
    else return(NULL)
```

Parse Tree for 2*3*4

Concrete
Parse Tree



Desired
Abstract
Parse Tree



Why Use Hand-Coded Parser?

- Why not use parser generator?
- What do you do if your parser doesn't work?
  - Recursive descent parser – write more code
  - Parser generator
    - Hack grammar
    - But if parser generator doesn't work, nothing you can do
- If you have complicated grammar
  - Increase chance of going outside comfort zone of parser generator
  - Your parser may NEVER work

Bottom Line

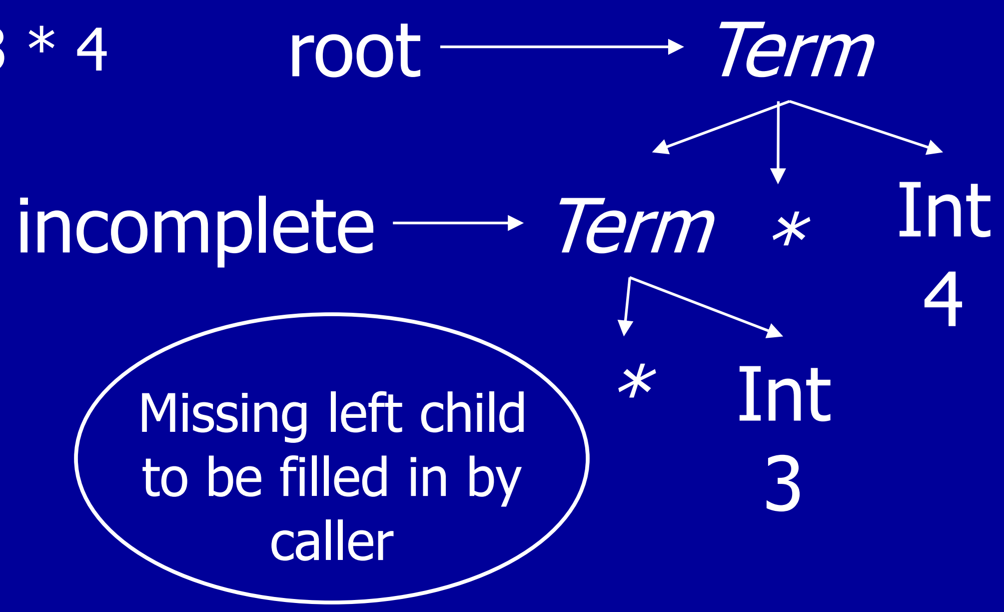
- Recursive descent parser properties
  - Probably more work
  - But less risk of a disaster - you can almost always make a recursive descent parser work
  - May have easier time dealing with resulting code
    - Single language system
    - No need to deal with potentially flaky parser generator
    - No integration issues with automatically generated code
- If your parser development time is small compared to rest of project, or you have a really complicated language, use hand-coded recursive descent parser

Summary

- Top-Down Parsing
- Use Lookahead to Avoid Backtracking
- Parser is
  - Hand-Coded
  - Set of Mutually Recursive Procedures

Direct Generation of Abstract Tree

- TermPrime builds an incomplete tree
  - Missing leftmost child
  - Returns root and incomplete node
- (root, incomplete) = TermPrime()
  - Called with token = *
  - Remaining tokens = 3 * 4



Code for Term

```
Term()
    if (token = Int n)
        leftmostInt = token; token = NextToken();
        (root, incomplete) = TermPrime();
        if (root == NULL) return leftmostInt;
        incomplete.leftChild = leftmostInt;
        return root;
    else throw SyntaxError
```

Input to parse

2*3*4

token $\longrightarrow$ Int 2

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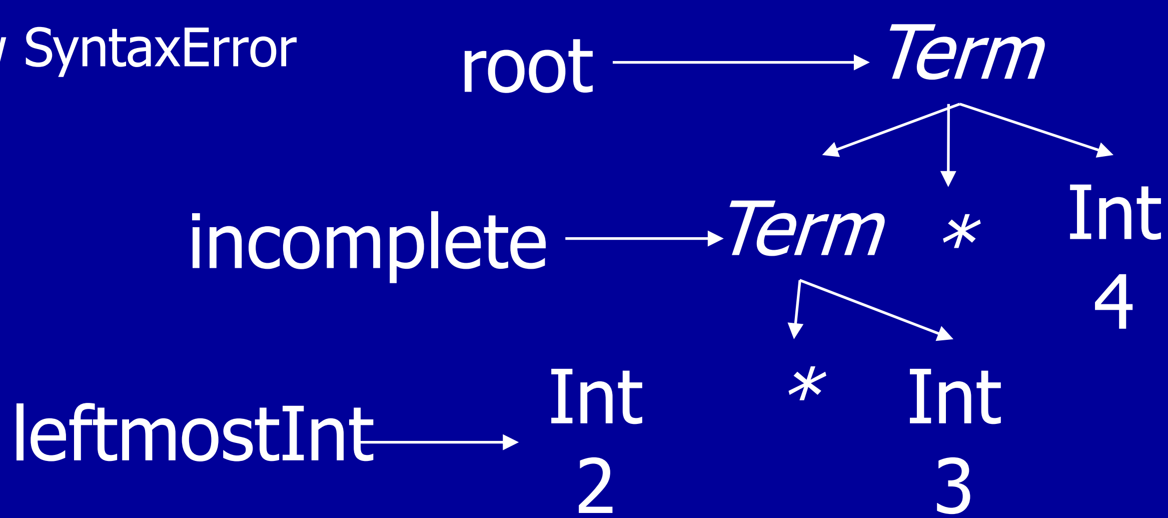

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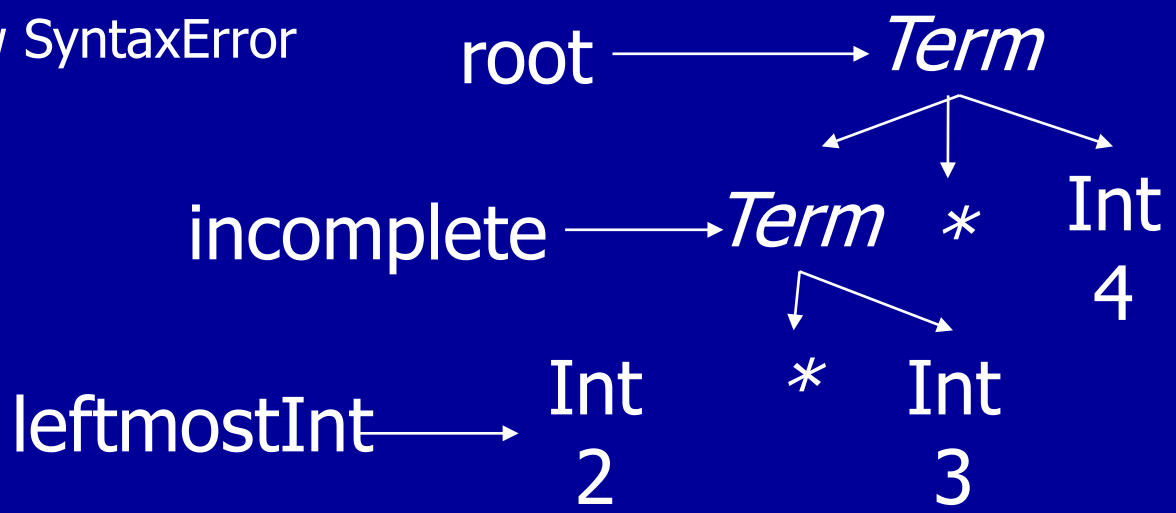

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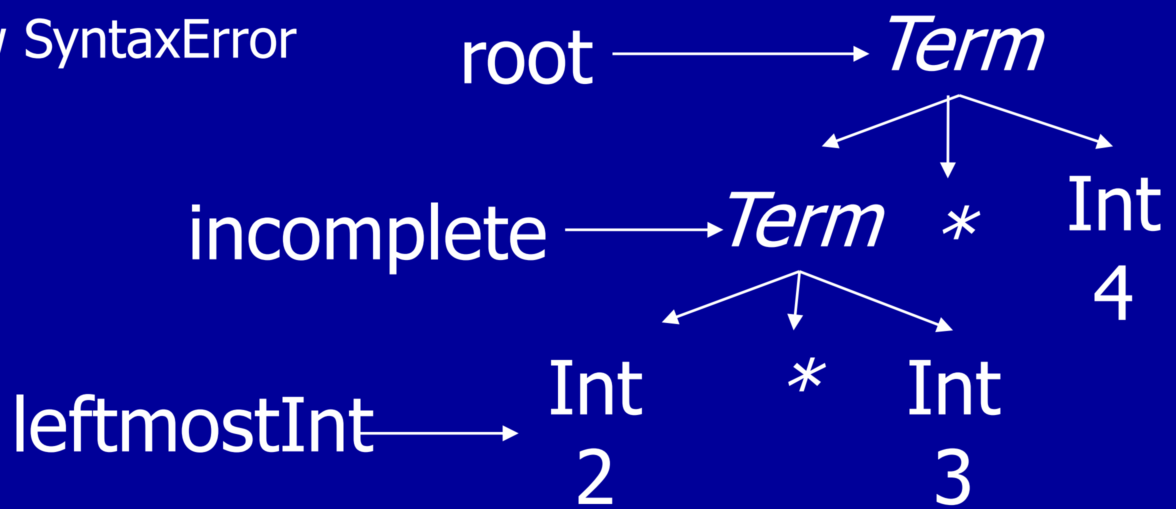

Code for TermPrime

```
TermPrime()
    if (token = *) || (token = /)
        op = token; next = NextToken();
        if (next = Int n)
            token = NextToken();
            (root, incomplete) = TermPrime();
            if (root == NULL)
                root = new ExprNode(NULL, op, next);
                return (root, root);
            else    newChild = new ExprNode(NULL, op, next);
                incomplete.leftChild = newChild;
                return(root, newChild);
        else throw SyntaxError
    else return(NULL,NULL)
```

Missing left child to be filled in by caller