6.035

Introduction to Program Analysis and Optimization

Outline

- Introduction
- Basic Blocks
- Common Subexpression Elimination
- Copy Propagation
- Dead Code Elimination
- Algebraic Simplification
- Summary

Program Analysis

- Compile-time reasoning about run-time behavior of program
  - Can discover things that are always true:
    - "x is always 1 in the statement y = x + z"
    - "the pointer p always points into array a"
    - "the statement return 5 can never execute"
  - Can infer things that are likely to be true:
    - "the reference r usually refers to an object of class C"
    - "the statement a = b + c appears to execute more frequently than the statement x = y + z"
  - Distinction between data and control-flow properties

Transformations

- Use analysis results to transform program
- Overall goal: improve some aspect of program
- Traditional goals:
  - Reduce number of executed instructions
  - Reduce overall code size
- Other goals emerge as space becomes more complex
  - Reduce number of cycles
    - Use vector or DSP instructions
    - Improve instruction or data cache hit rate
  - Reduce power consumption
  - Reduce memory usage

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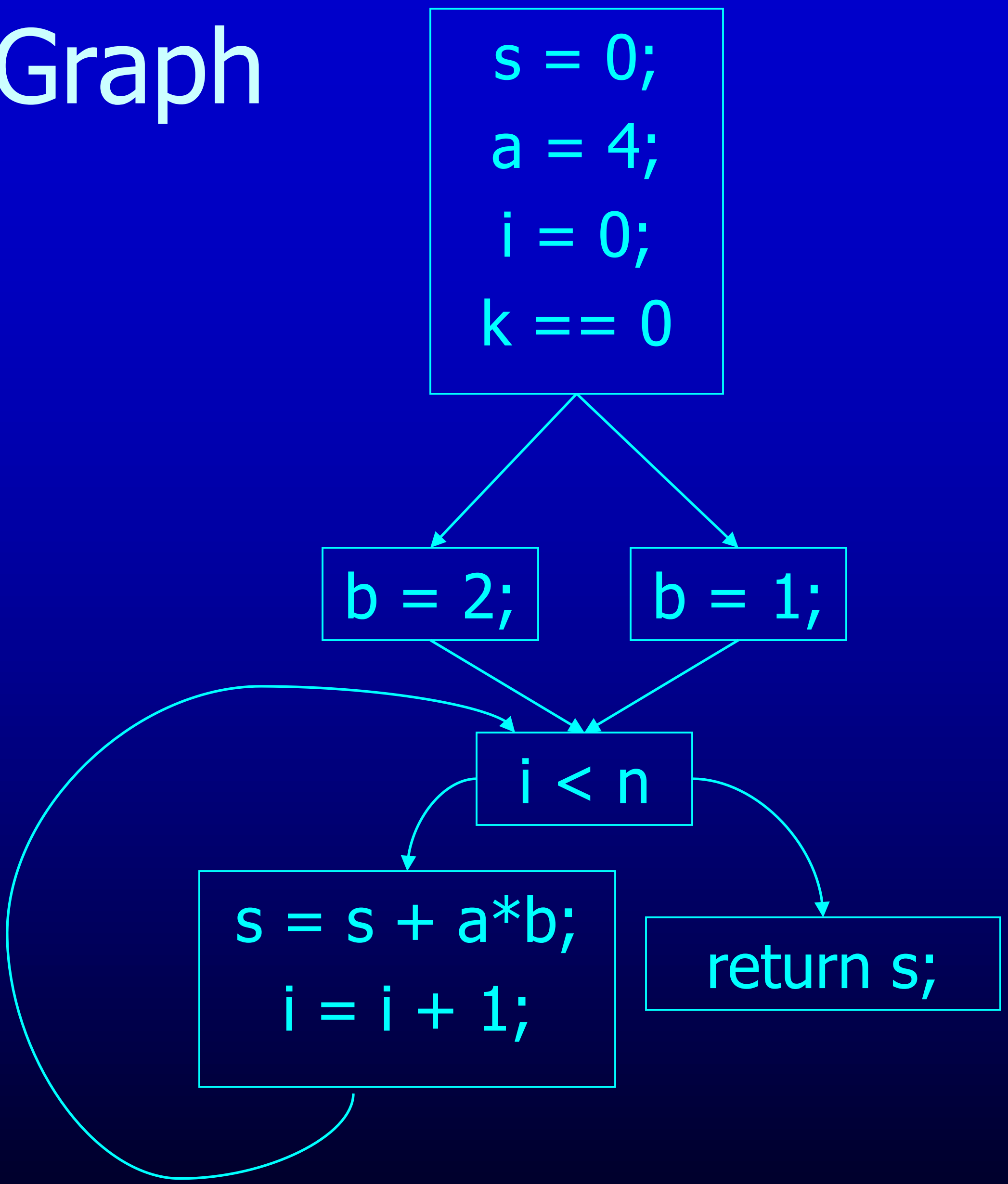
Control Flow Graph

- Nodes Represent Computation
  - Each Node is a Basic Block
  - Basic Block is a Sequence of Instructions with
    - No Branches Out Of Middle of Basic Block
    - No Branches Into Middle of Basic Block
    - Basic Blocks should be maximal
  - Execution of basic block starts with first instruction
  - Includes all instructions in basic block
- Edges Represent Control Flow

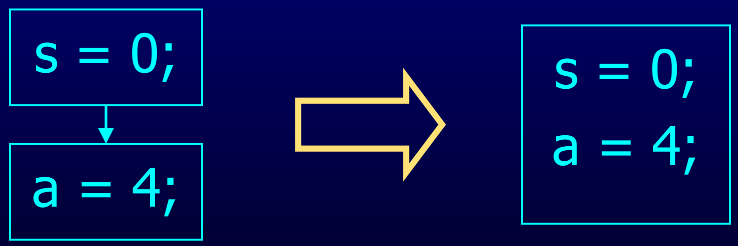
Control Flow Graph

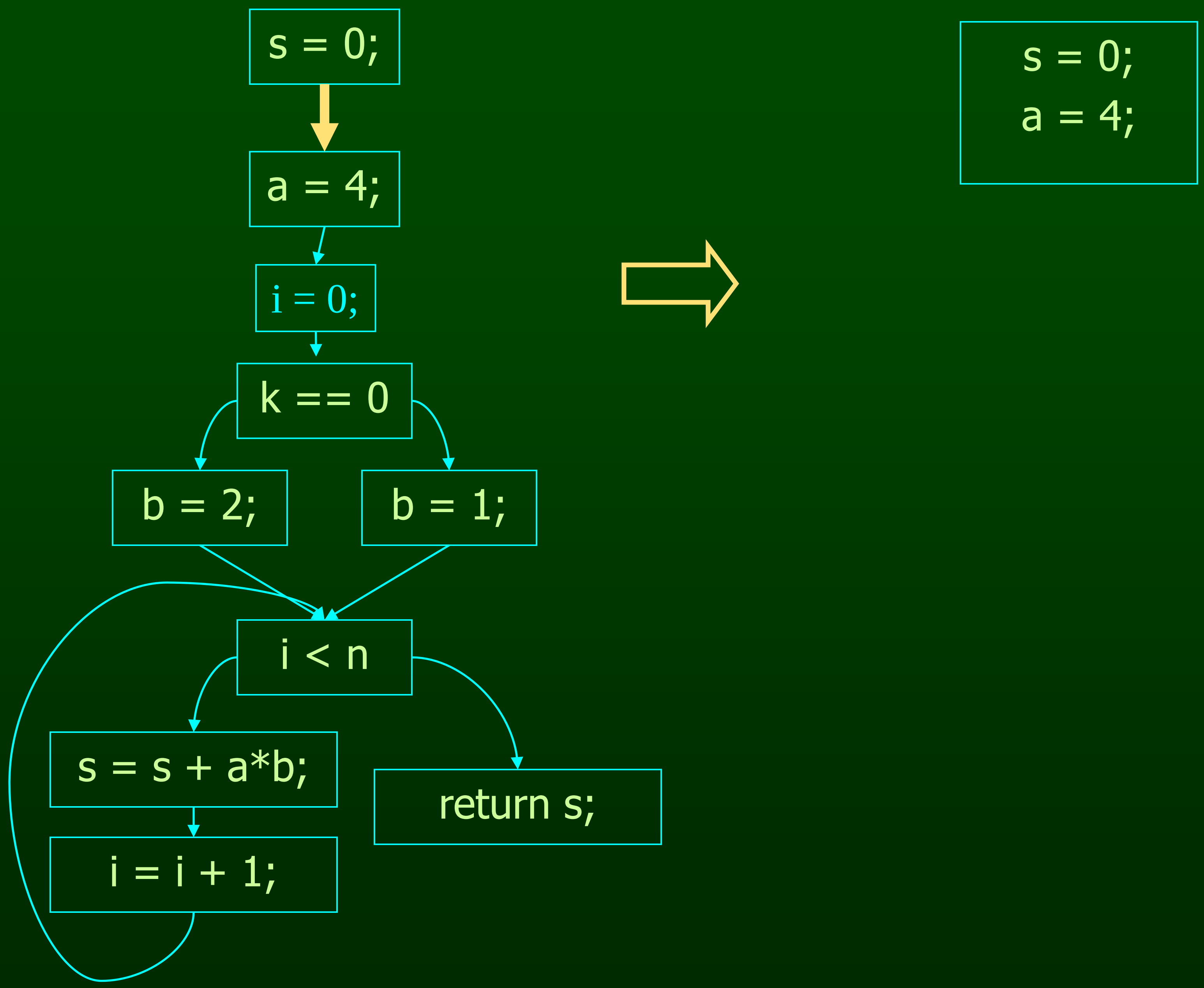
```
into add(n, k) {
    s = 0; a = 4; i = 0;
    if (k == 0)
        b = 1;
    else
        b = 2;
    while (i < n) {
        s = s + a*b;
        i = i + 1;
    }
    return s;
}
```



Basic Block Construction

- Start with instruction control-flow graph
- Visit all edges in graph
- Merge adjacent nodes if
  - Only one edge from first node
  - Only one edge into second node





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i = i + 1;
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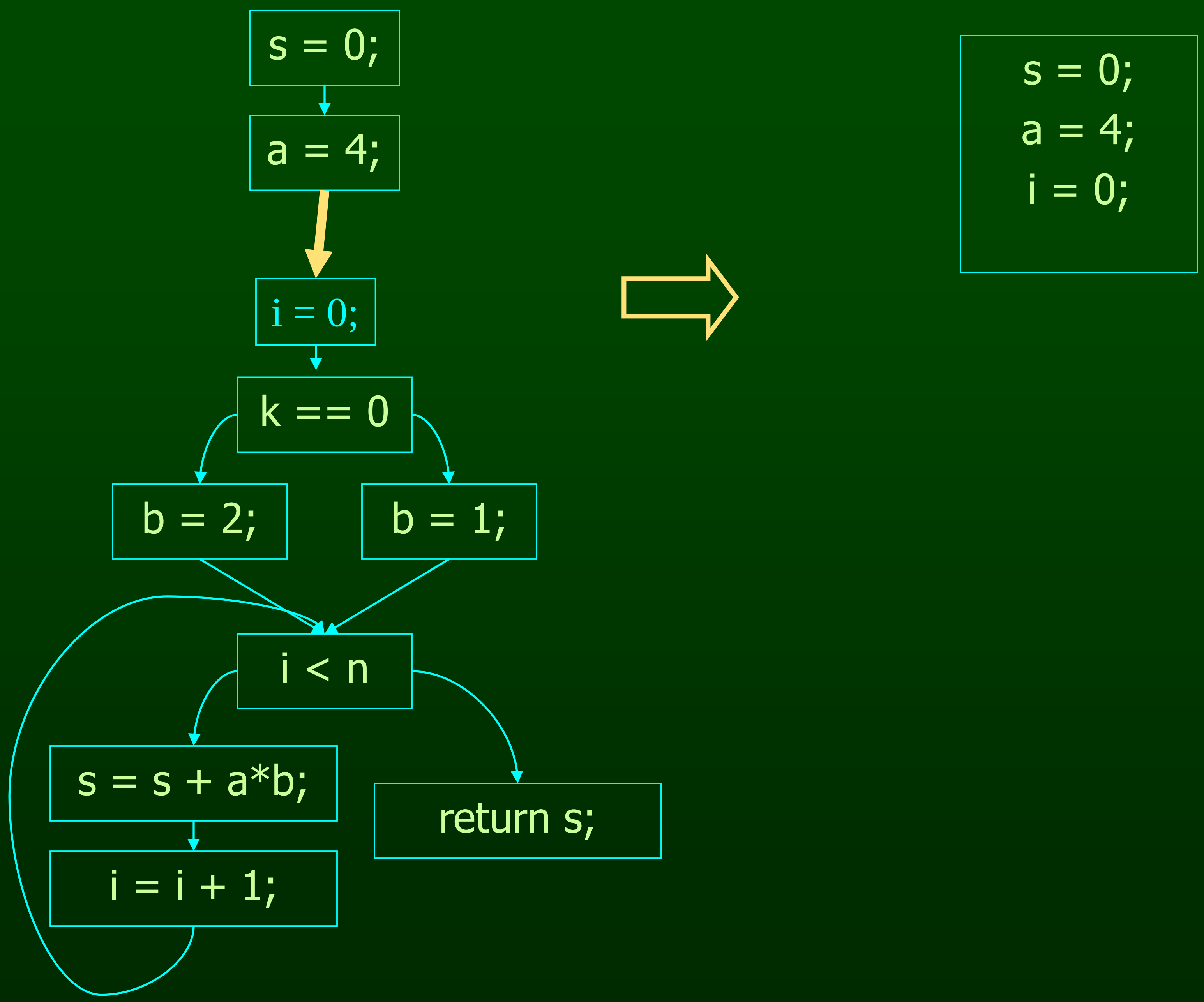
a = 4;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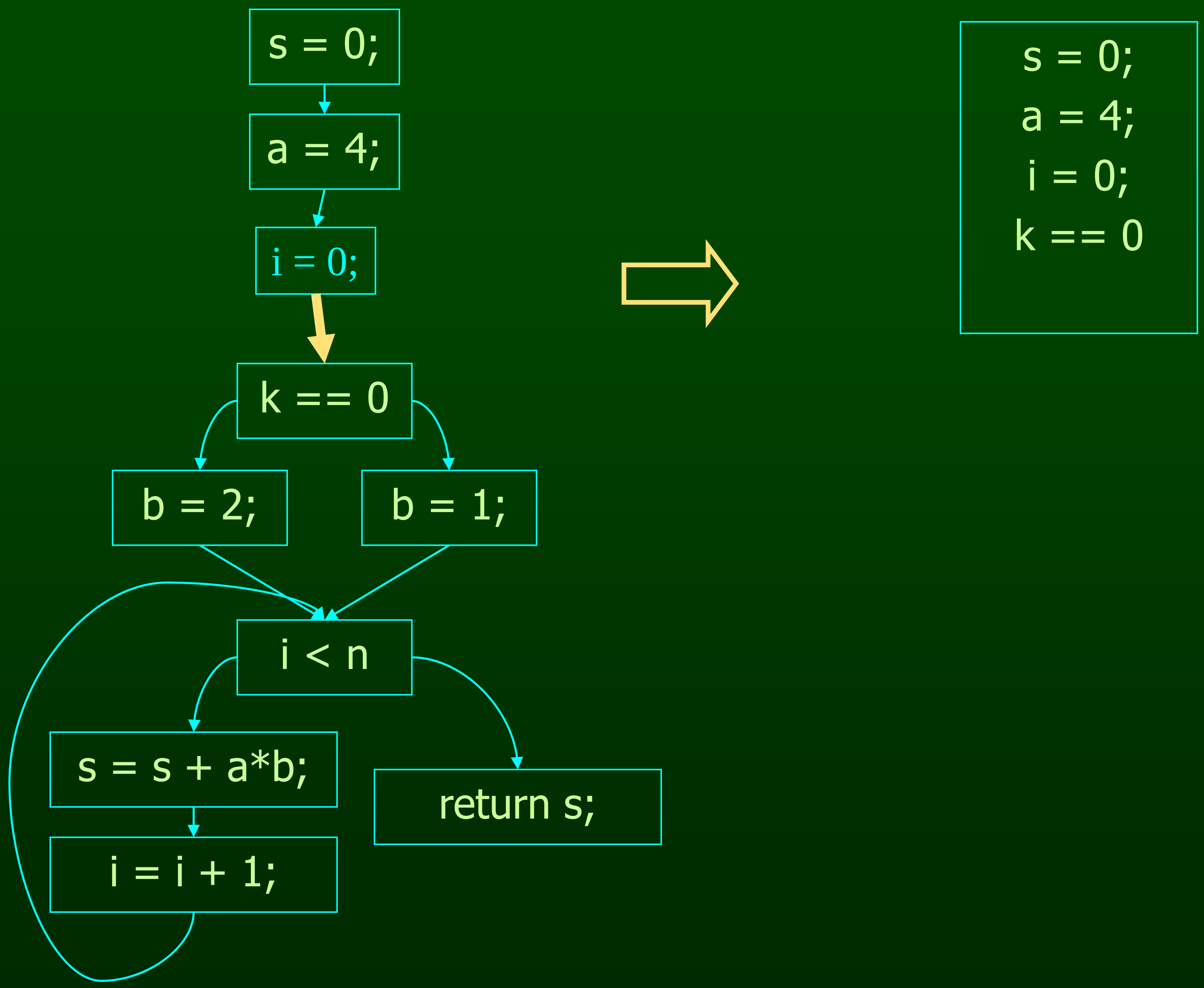
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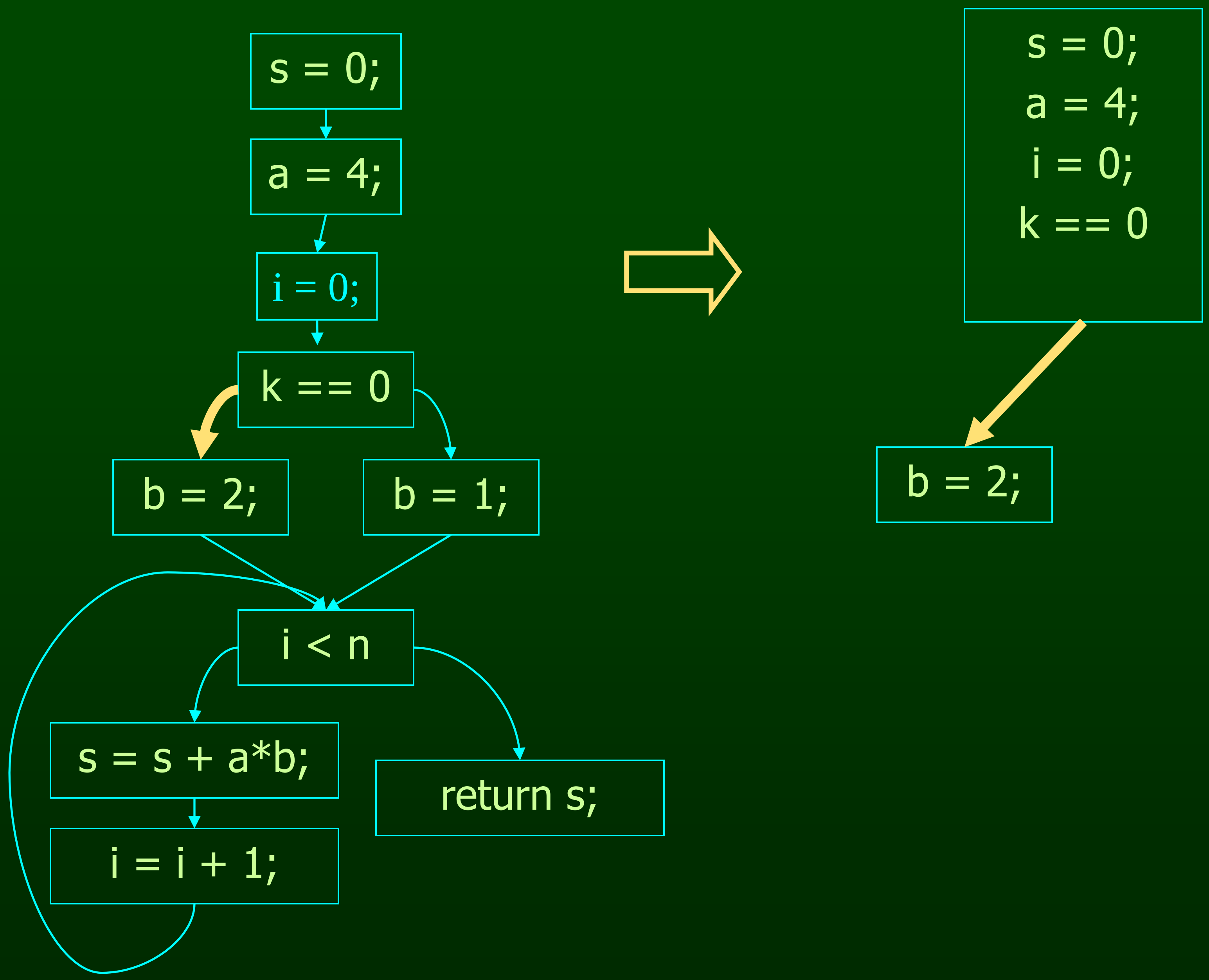
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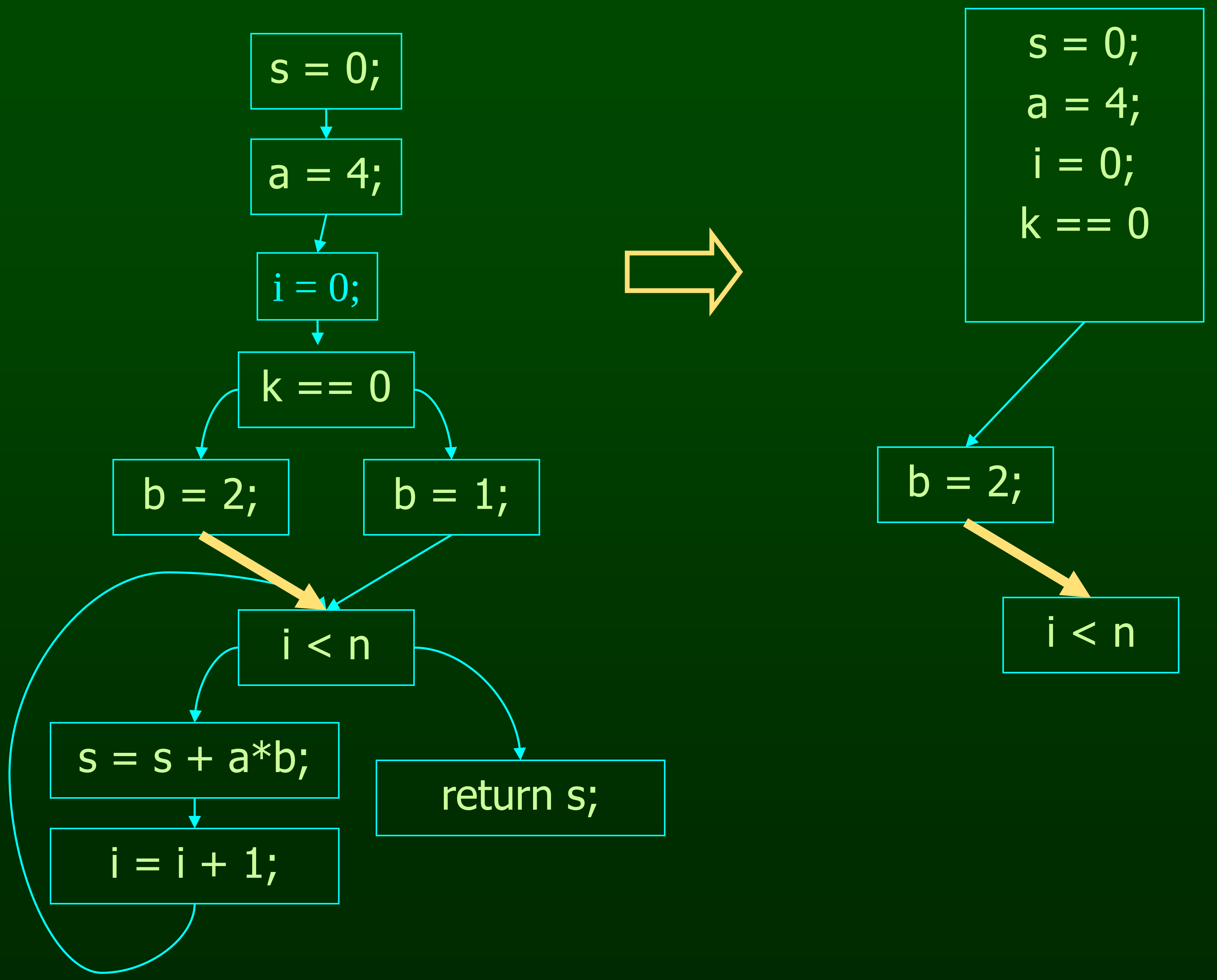
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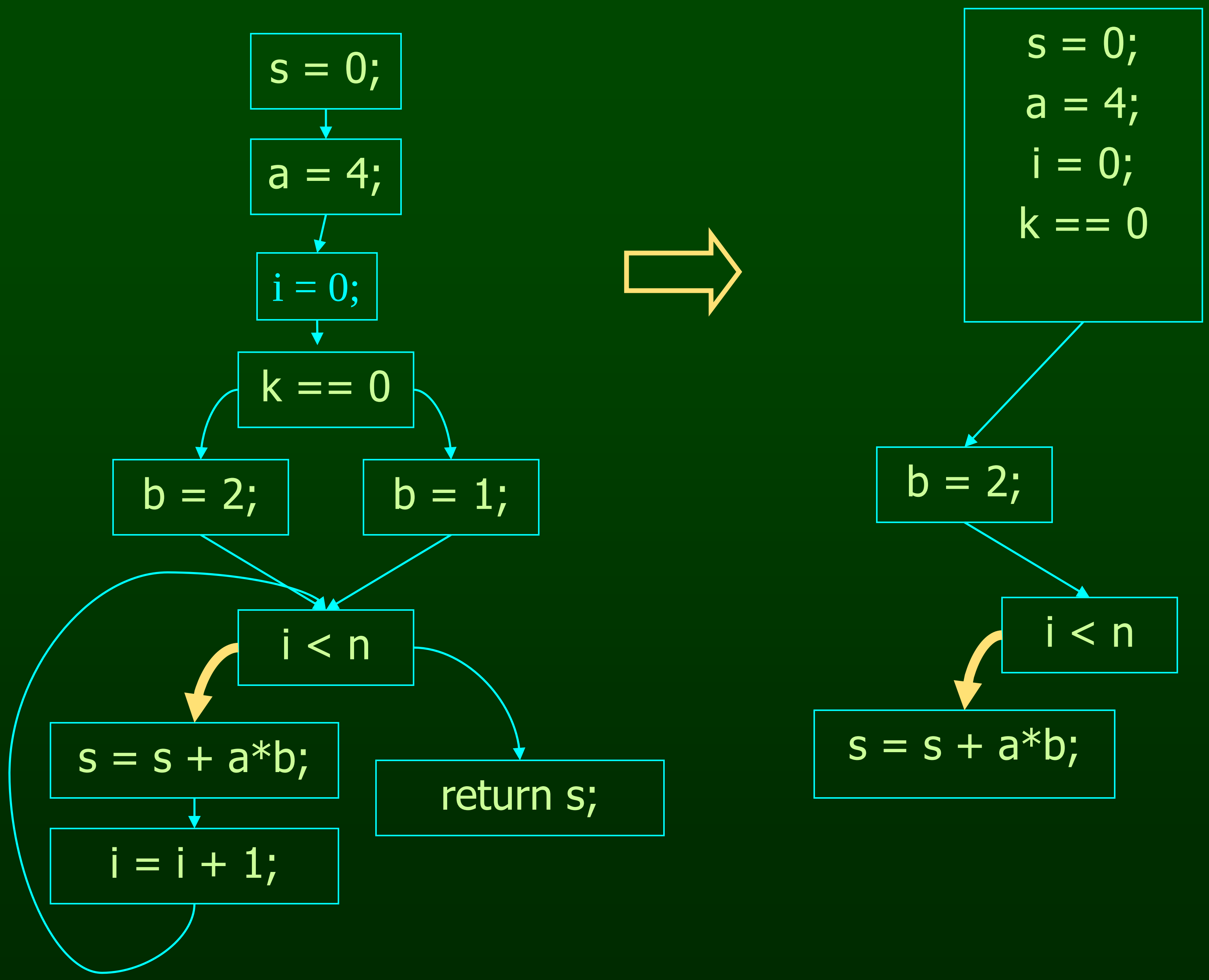

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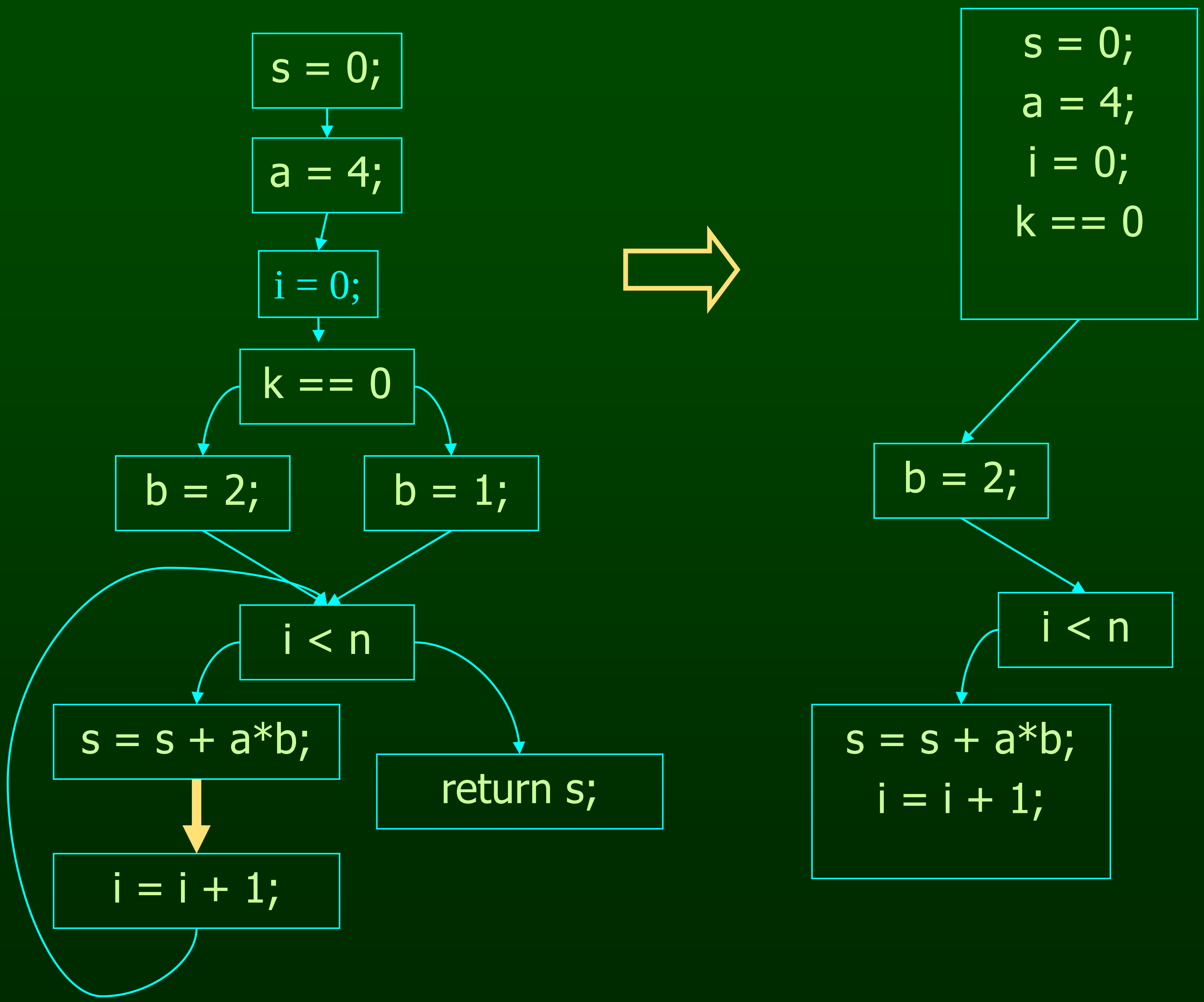
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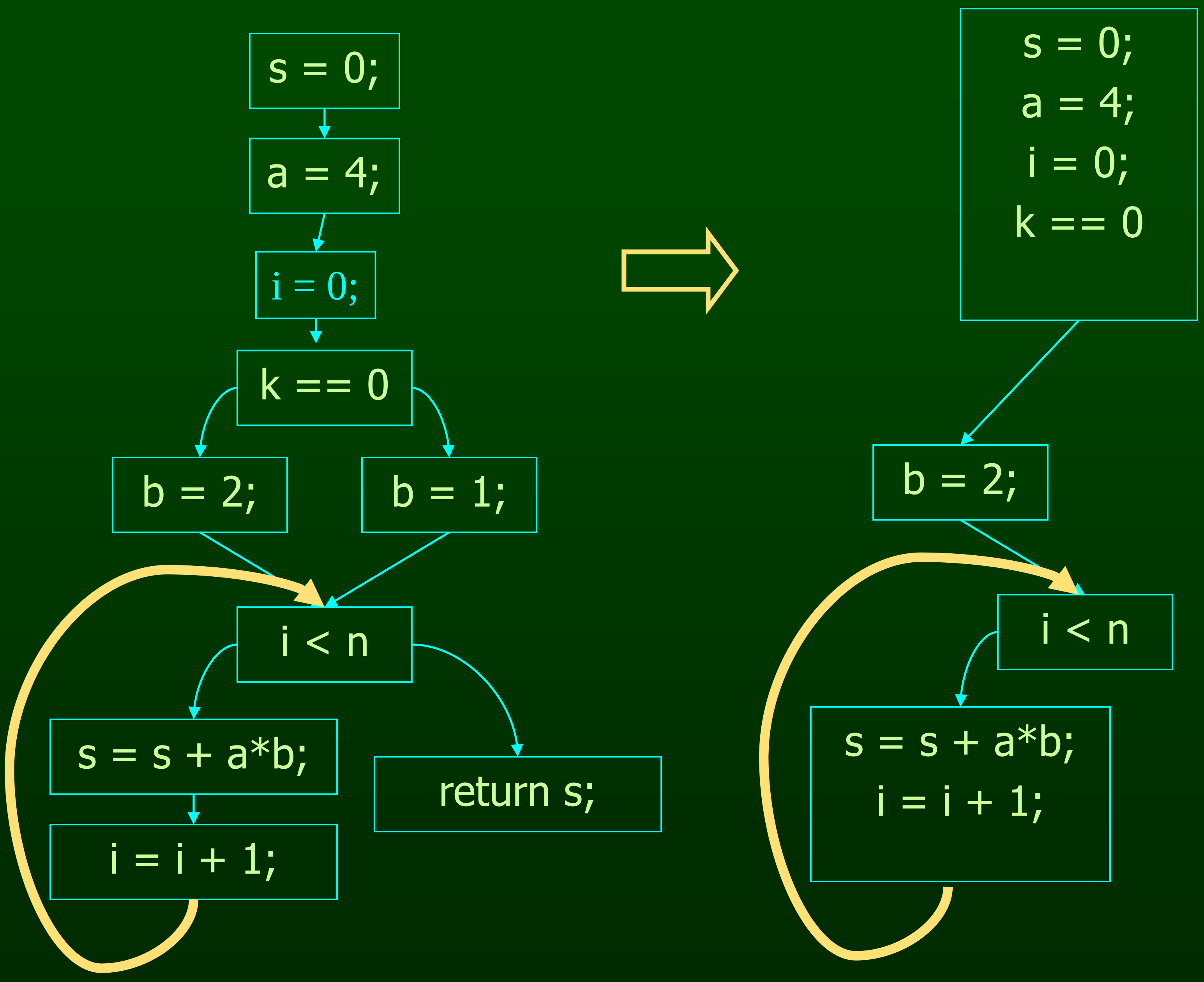

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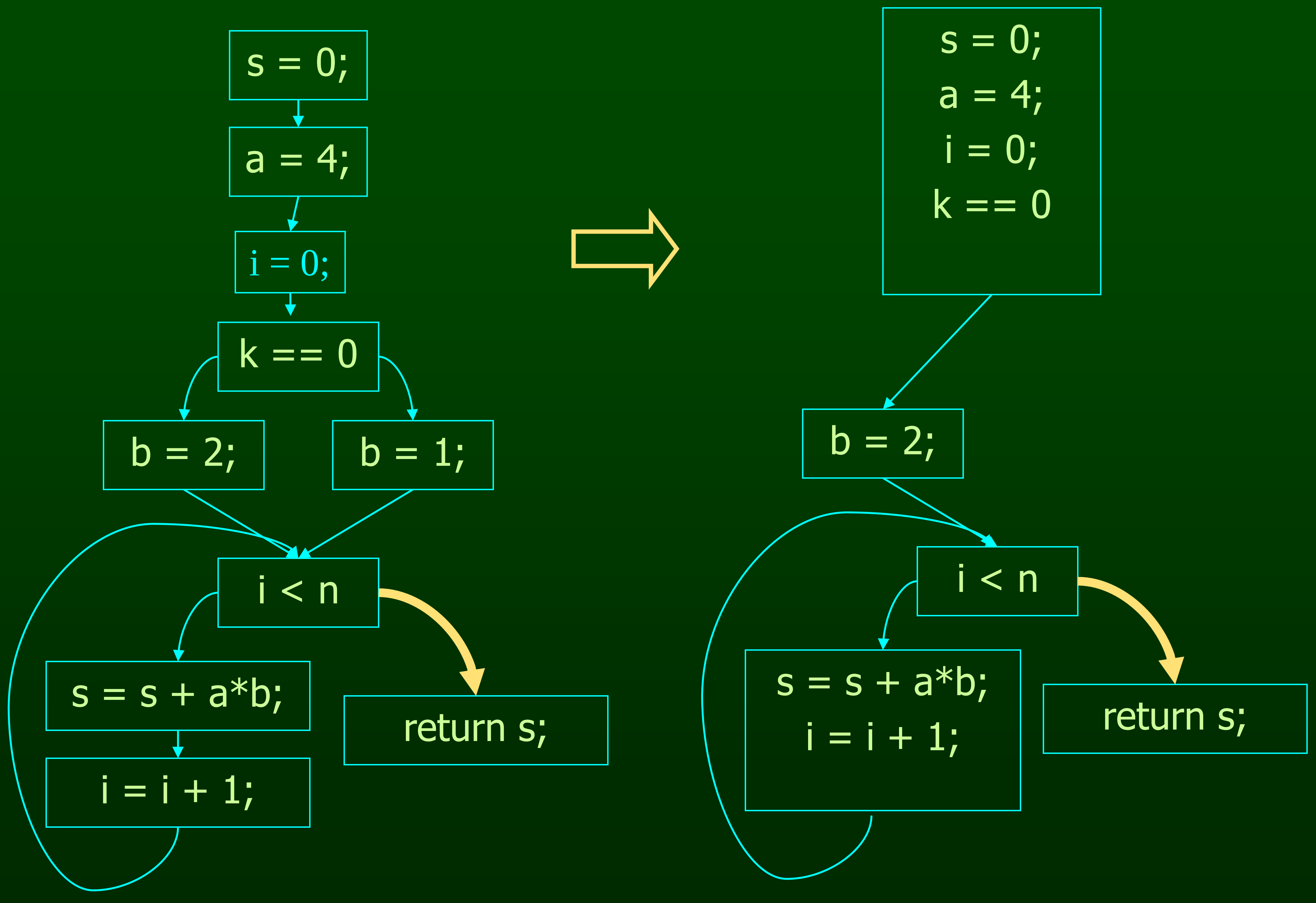

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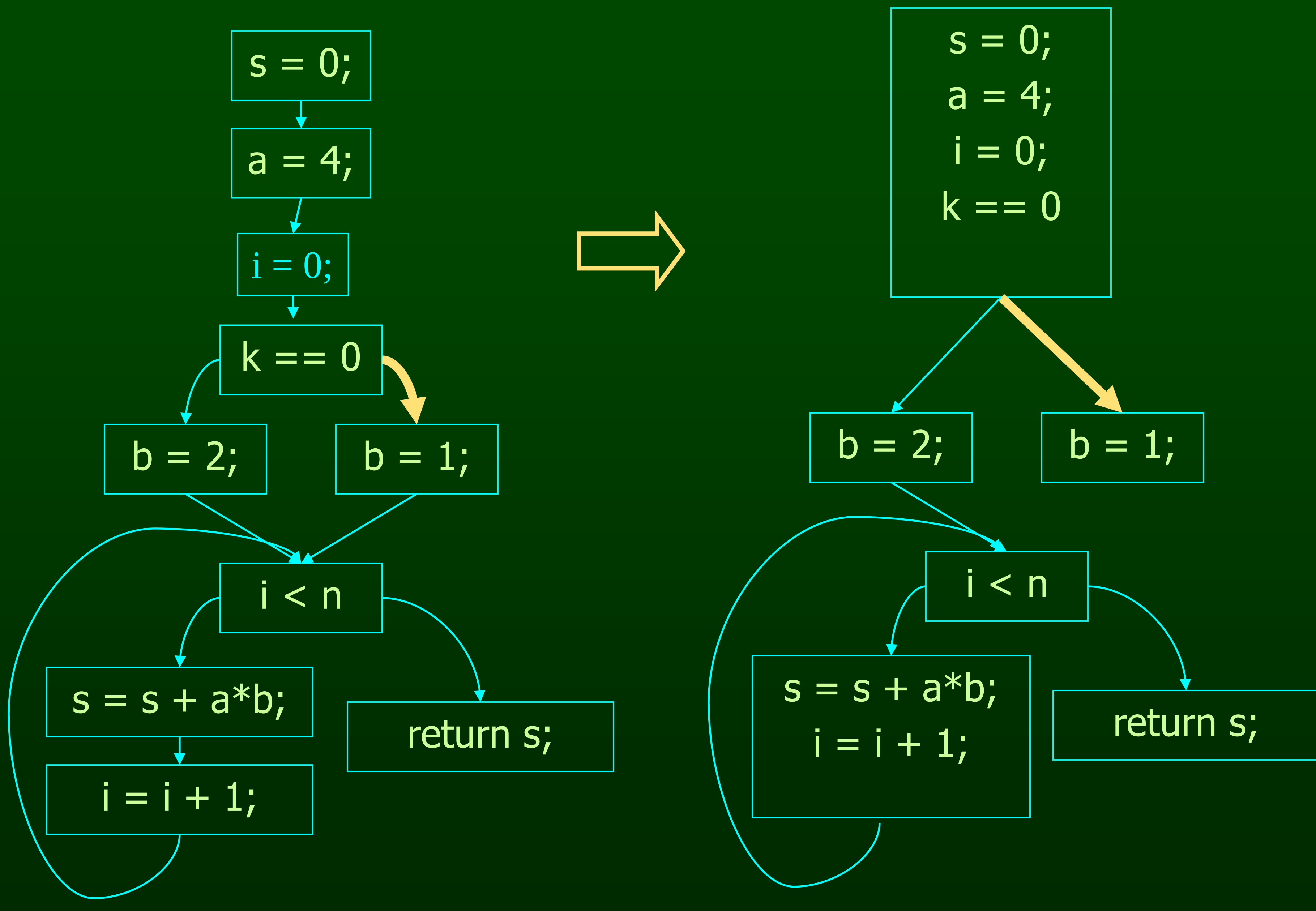

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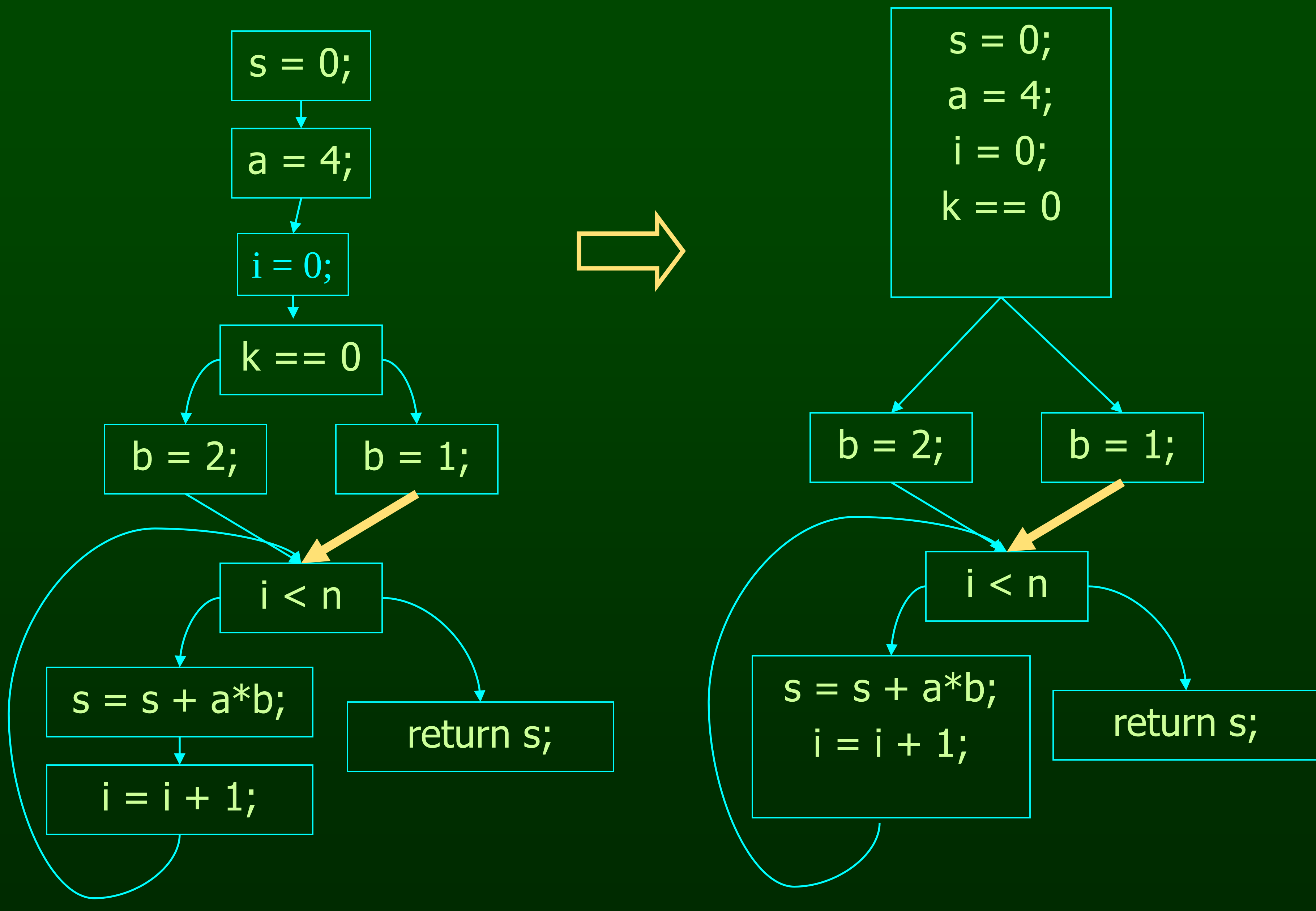

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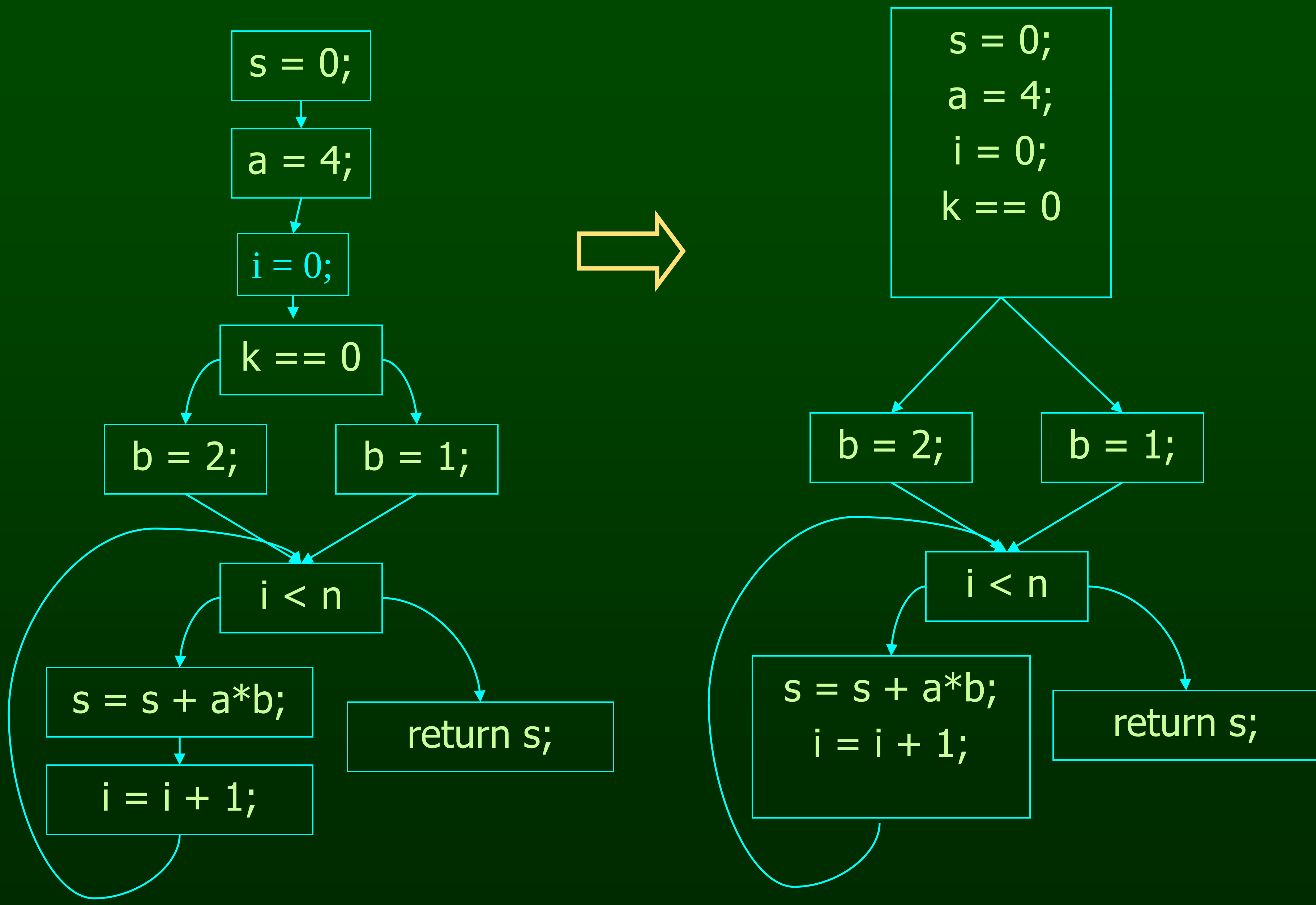
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Program Points, Split and Join Points

- One program point before and after each statement in program
- Split point has multiple successors – conditional branch statements only split points
- Merge point has multiple predecessors
- Each basic block
  - Either starts with a merge point or its predecessor ends with a split point
  - Either ends with a split point or its successor starts with a merge point

Basic Block Optimizations

- Common Sub-Expression Elimination
  - a=(x+y)+z; b=x+y;
  - t=x+y; a=t+z; b=t;
- Constant Propagation
  - x=5; b=x+y;
  - x=5; b=5+y;
- Algebraic Identities
  - a=x*1;
  - a=x;
- Copy Propagation
  - a=x+y; b=a; c=b+z;
  - a=x+y; b=a; c=a+z;
- Dead Code Elimination
  - a=x+y; b=a; b=a+z;
  - a=x+y;   b=a+z
- Strength Reduction
  - t=i*4;
  - t=i<<2;

Basic Block Analysis Approach

- Assume normalized basic block - all statements are of the form
  - var = var op var (where op is a binary operator)
  - var = op var (where op is a unary operator)
  - var = var
- Simulate a symbolic execution of basic block
  - Reason about values of variables (or other aspects of computation)
  - Derive property of interest

Two Kinds of Variables

- Temporaries Introduced By Compiler
  - Transfer values only within basic block
  - Introduced as part of instruction flattening
  - Introduced by optimizations/transformations
  - Typically assigned to only once
- Program Variables
  - Declared in original program
  - May be assigned to multiple times
  - May transfer values between basic blocks

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Value Numbering

- Reason about values of variables and expressions in the program
  - Simulate execution of basic block
  - Assign virtual value to each variable and expression

- Discovered property: which variables and expressions have the same value

- Standard use:
  - Common subexpression elimination
  - Typically combined with transformation that
    - Saves computed values in temporaries
    - Replaces expressions with temporaries when value of expression previously computed

Original Basic Block

a = x+y
b = a+z
b = b+y
c = a+z

New Basic Block

a = x+y
t1 = a
b = a+z
t2 = b
b = b+y
t3 = b
c = t2

Var to Val

x → v1
y → v2
a → v3
z → v4
b → v6
c → v5

Exp to Val

v1+v2 → v3
v3+v4 → v5
v5+v2 → v6

Exp to Tmp

v1+v2 → t1
v3+v4 → t2
v5+v2 → t3

Value Numbering Summary

- Forward symbolic execution of basic block
- Each new value assigned to temporary
  - a=x+y; becomes a=x+y; t=a;
  - Temporary preserves value for use later in program even if original variable rewritten
    - a=x+y;   a=a+z; b=x+y becomes
    - a=x+y; t=a; a=a+z; b=t;
- Maps
  - Var to Val – specifies symbolic value for each variable
  - Exp to Val – specifies value of each evaluated expression
  - Exp to Tmp – specifies tmp that holds value of each evaluated expression

Map Usage

- Var to Val
  - Used to compute symbolic value of y and z when processing statement of form x = y + z
- Exp to Tmp
  - Used to determine which tmp to use if value(y) + value(z) previously computed when processing statement of form x = y + z
- Exp to Val
  - Used to update Var to Val when
    - processing statement of the form x = y + z, and
    - value(y) + value(z) previously computed

Interesting Properties

- Finds common subexpressions even if they use different variables in expressions
  - y=a+b; x=b; z=a+x becomes
  - y=a+b; t=y; x=b; z=t
  - Why? Because computes with symbolic values
- Finds common subexpressions even if variable that originally held the value was overwritten
  - y=a+b; y=1; z=a+b becomes
  - y=a+b; t=y; y=1; z=t
  - Why? Because saves values away in temporaries

One More Interesting Property

- Flattening and CSE combine to capture partial and arbitrarily complex common subexpressions

  w=(a+b)+c; x=b; y=(a+x)+c; z=a+b;

  - After flattening:

  t1=a+b; w=t1+c; x=b; t2=a+x; y=t2+c; z=a+b;

  - CSE algorithm notices that
    - t1+c and t2+c compute same value
    - In the statement z = a+b, a+b has already been computed so generated code can reuse the result

  t1=a+b; w=t1+c; t3=w; x=b; t2=t1; y=t3; z=t1;

Problems I

- Algorithm has a temporary for each new value
  - a=x+y; t1=a;
- Introduces
  - lots of temporaries
  - lots of copy statements to temporaries
- In many cases, temporaries and copy statements are unnecessary
- So we eliminate them with copy propagation and dead code elimination

Problems II

- Expressions have to be identical
  - a=x+y+z; b=y+z+x; c=x*2+y+2*z–(x+z)
- We use canonicalization
- We use algebraic simplification

Copy Propagation

- Once again, simulate execution of program
- If can, use original variable instead of temporary
  - a=x+y; b=x+y;
  - After CSE becomes a=x+y; t=a; b=t;
  - After CP becomes a=x+y; t=a; b=a;
  - After DCE becomes a=x+y; b=a;
- Key idea:
  - determine when original variable is NOT overwritten between its assignment statement and the use of the computed value
  - If not overwritten, use original variable

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Copy Propagation Maps

- Maintain two maps
  - tmp to var: tells which variable to use instead of a given temporary variable
  - var to set: inverse of tmp to var. tells which temps are mapped to a given variable by tmp to var

Copy Propagation Example

- Original

  a = x+y

  b = a+z

  c = x+y

  a = b

- After CSE

  a = x+y

  t1 = a

  b = a+z

  t2 = b

  c = t1

  a = b

- After CSE and Copy Propagation

  a = x+y

  t1 = a

  b = a+z

  t2 = b

  c = a

  a = b

Copy Propagation Example

Basic Block After CSE

a = x+y
t1 = a

Basic Block After CSE and Copy Prop

a = x+y
t1 = a

tmp to var

$t1 \rightarrow a$

var to set

$a \rightarrow \{t1\}$

Copy Propagation Example

Basic Block
After CSE

```
a = x+y
t1 = a
b = a+z
t2 = b
```

Basic Block After
CSE and Copy Prop

```
a = x+y
t1 = a
b = a+z
t2 = b
```

tmp to var

$t1 \rightarrow a$
$t2 \rightarrow b$

var to set

$a \rightarrow \{t1\}$
$b \rightarrow \{t2\}$

Copy Propagation Example

Basic Block After CSE

```
a = x+y
t1 = a
b = a+z
t2 = b
c = t1
```

tmp to var

$t1 \rightarrow a$

$t2 \rightarrow b$

Basic Block After CSE and Copy Prop

```
a = x+y
t1 = a
b = a+z
t2 = b
```

var to set

$a \rightarrow \{t1\}$

$b \rightarrow \{t2\}$

Copy Propagation Example

Basic Block After CSE

a = x+y
t1 = a
b = a+z
t2 = b
c = t1

tmp to var

t1 → a
t2 → b

Basic Block After CSE and Copy Prop

a = x+y
t1 = a
b = a+z
t2 = b
c = a

var to set

a →{t1}
b →{t2}

Copy Propagation Example

Basic Block After CSE

a = x+y
t1 = a
b = a+z
t2 = b
c = t1
a = b

tmp to var

t1 → a
t2 → b

Basic Block After CSE and Copy Prop

a = x+y
t1 = a
b = a+z
t2 = b
c = a
a = b

var to set

a →{t1}
b →{t2}

Copy Propagation Example

Basic Block After CSE

a = x+y
t1 = a
b = a+z
t2 = b
c = t1
a = b

tmp to var

t1 → t1
t2 → b

Basic Block After CSE and Copy Prop

a = x+y
t1 = a
b = a+z
t2 = b
c = a
a = b

var to set

a → {}
b → {t2}

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Dead Code Elimination

- Copy propagation keeps all temps around
- May be temps that are never read
- Dead Code Elimination removes them

Basic Block After CSE and CP

```
a = x+y
t1 = a
b = a+z
t2 = b
c = a
a = b
```

Basic Block After CSE, CP and DCE

```
a = x+y
b = a+z
c = a
a = b
```

Dead Code Elimination

- Basic Idea
  - Process Code In Reverse Execution Order
  - Maintain a set of variables that are needed later in computation
  - If encounter an assignment to a temporary that is not needed, remove assignment

Basic Block After CSE and Copy Prop

```
a = x+y
t1 = a
b = a+z
t2 = b
c = a
a = b
```

Needed Set

{b}

Basic Block After CSE and Copy Prop

```
a = x+y
t1 = a
b = a+z
t2 = b
c = a
a = b
```

Needed Set

{a, b}

Basic Block After CSE and Copy Prop

```
a = x+y
t1 = a
b = a+z
t2 = b
c = a
a = b
```

Needed Set

{a, b}

Basic Block After CSE and Copy Prop

```
a = x+y
t1 = a
b = a+z
⇨
c = a
a = b
```

Needed Set

{a, b}

Basic Block After CSE and Copy Prop

```
a = x+y
t1 = a
b = a+z

c = a
a = b
```

Needed Set

{a, z}

Basic Block After CSE and Copy Prop

```
a = x+y
t1 = a
b = a+z

c = a
a = b
```

Needed Set

{a, z}

Basic Block After CSE and Copy Prop

a = x+y

b = a+z

c = a
a = b

Needed Set
{a, z}

Basic Block After , CSE Copy Propagation, and Dead Code Elimination

```
a = x+y

b = a+z

c = a
a = b
```

Needed Set

{x, y, z}

Basic Block After , CSE Copy Propagation, and Dead Code Elimination

```
a = x+y

b = a+z

c = a
a = b
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Needed Set

{x, y, z}

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Algebraic Simplification

- Apply our knowledge from algebra, number theory etc. to simplify expressions

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- Example
  - a + 0 $\Rightarrow$ a
  - a * 1 $\Rightarrow$ a
  - a / 1 $\Rightarrow$ a
  - a * 0 $\Rightarrow$ 0
  - 0 - a $\Rightarrow$ -a
  - a + (-b) $\Rightarrow$ a - b
  - -(-a) $\Rightarrow$ a

Algebraic Simplification

- Apply our knowledge from algebra, number theory etc. to simplify expressions
- Example
  - $a \wedge \text{true} \Rightarrow a$
  - $a \wedge \text{false} \Rightarrow \text{false}$
  - $a \vee \text{true} \Rightarrow \text{true}$
  - $a \vee \text{false} \Rightarrow a$

Algebraic Simplification

- Apply our knowledge from algebra, number theory etc. to simplify expressions
- Example
  - a ^ 2 $\Rightarrow$ a*a
  - a * 2 $\Rightarrow$ a + a
  - a * 8 $\Rightarrow$ a << 3

Opportunities for Algebraic Simplification

- In the code
  - Programmers are lazy to simplify expressions
  - Programs are more readable with full expressions
- After compiler expansion
  - Example: Array read A[8][12] will get expanded to
  - *(Abase + 4*(12 + 8*256)) which can be simplified
- After other optimizations

Usefulness of Algebraic Simplification

- Reduces the number of instructions
- Uses less expensive instructions
- Enable other optimizations

Implementation

- Not a data-flow optimization!
- Find candidates that matches the simplification rules and simplify the expression trees
- Candidates may not be obvious

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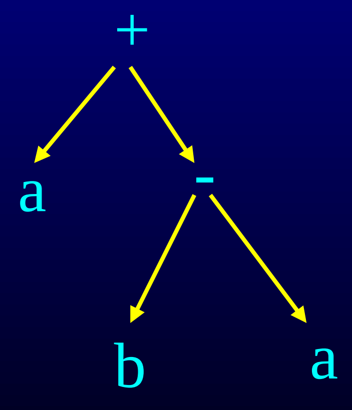
- Candidates may not be obvious
  - Example
    a + b - a



Use knowledge about operators

- Commutative operators
  - a **op** b = b **op** a
  -
- Associative operators
  - (a **op** b) **op** c = b **op** (a **op** c)

Canonical Format

- Put expression trees into a canonical format
  - Sum of multiplicands
  - Variables/terms in a canonical order
  - Example
    (a+3)*(a+8)*4 $\Rightarrow$ 4*a*a+44*a+96
  - Section 12.3.1 of whale book talks about this

Effects on the Numerical Stability

- Some algebraic simplifications may produce incorrect results

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- Example
  - (a / b)*0 + c

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- Example
  - (a / b)*0 + c
  - we can simplify this to c

Effects on the Numerical Stability

- Some algebraic simplifications may produce incorrect results
- Example
  - (a / b)*0 + c
  - we can simplify this to c
  - But what about when b = 0
    should be a exception, but we'll get a result!

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Interesting Properties

- Analysis and Transformation Algorithms Symbolically Simulate Execution of Program
  - CSE and Copy Propagation go forward
  - Dead Code Elimination goes backwards
- Transformations stacked
  - Group of basic transformations work together
  - Often, one transformation creates inefficient code that is cleaned up by following transformations
  - Transformations can be useful even if original code may not benefit from transformation

Other Basic Block Transformations

- Constant Propagation
- Strength Reduction
  - a<<2 = a*4; a+a+a = 3*a;
- Do these in unified transformation framework, not in earlier or later phases

Summary

- Basic block analyses and transformations
- Symbolically simulate execution of program
  - Forward (CSE, copy prop, constant prop)
  - Backward (Dead code elimination)
- Stacked groups of analyses and transformations that work together
  - CSE introduces excess temporaries and copy statements
  - Copy propagation often eliminates need to keep temporary variables around
  - Dead code elimination removes useless code
- Similar in spirit to many analyses and transformations that operate across basic blocks