6.035

Introduction to Dataflow Analysis

Value Numbering Summary

- Forward symbolic execution of basic block
- Maps
  - Var2Val – symbolic value for each variable
  - Exp2Val – value of each evaluated expression
  - Exp2Tmp – tmp that holds value of each evaluated expression
- Algorithm
  - For each statement
    - If variables in RHS not in the Var2Val add it with a new value
    - If RHS expression in Exp2Tmp use that Temp
    - If not add RHS expression to Exp2Val with new value
    - Copy the value into a new tmp and add to EXp2Tmp

Copy Propagation Summary

- Forward Propagation within basic block
- Maps
  - tmp2var: tells which variable to use instead of a given temporary variable
  - var2set: inverse of tmp to var. tells which temps are mapped to a given variable by tmp to var
- Algorithm
  - For each statement
    - If any tmp variable in the RHS is in tmp2var replace it with var
    - If LHS var in var2set remove the variables in the set in tmp2var

Dead Code Elimination Summary

- Backward Propagation within basic block
- Map
  - A set of variables that are needed later in computation
- Algorithm
  - Every statement encountered
    - If LHS is not in the set, remove the statement
    - Else put all the variables in the RHS into the set

Summary So far… what's next

- Till now: How to analyze and transform within a basic block

- Next: How to do it for the entire procedure

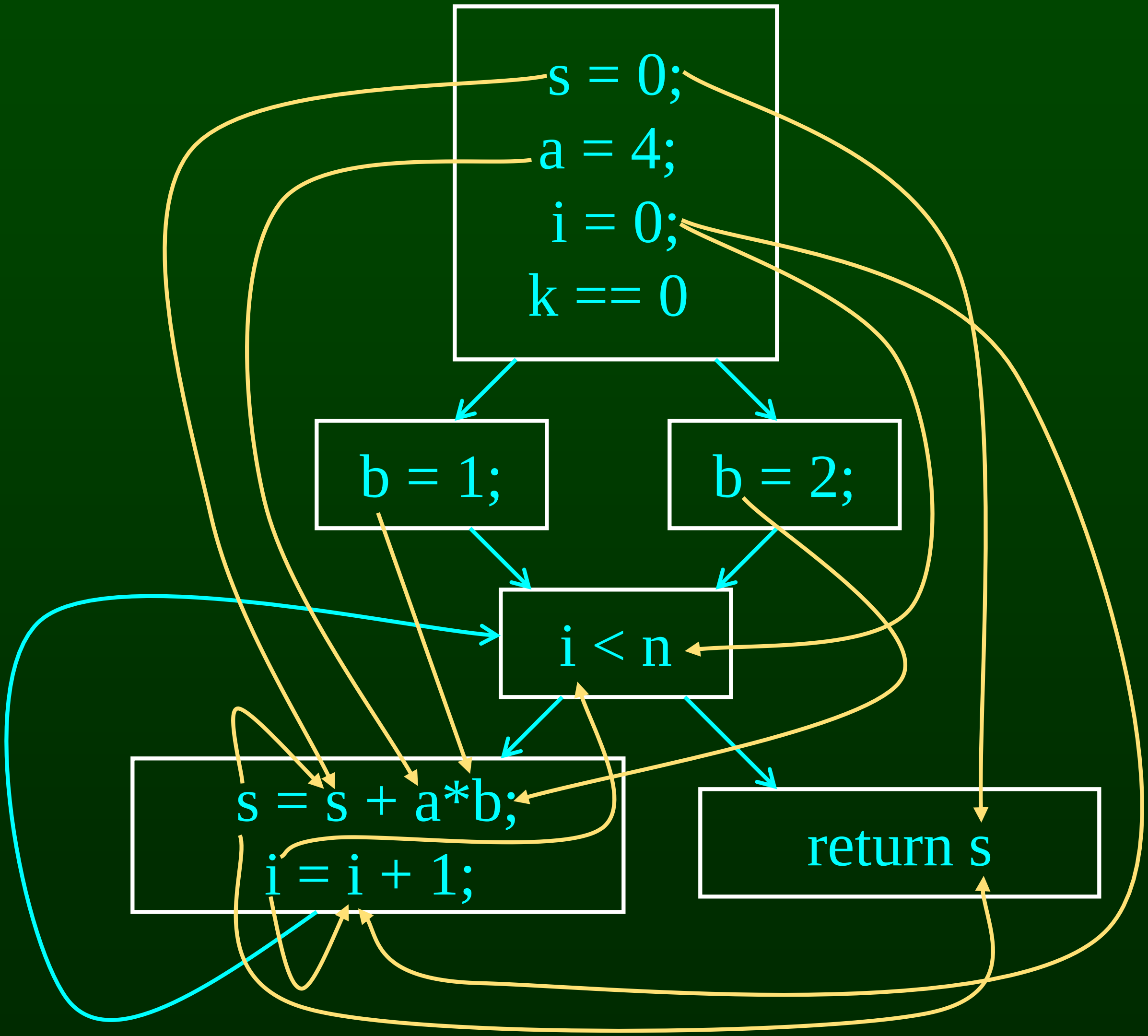
Outline

- Reaching Definitions
- Available Expressions
- Liveness

Reaching Definitions

- Concept of definition and use
  - a = x+y
  - is a definition of a
  - is a use of x and y
- A definition reaches a use if
  - value written by definition
  - may be read by use

Reaching Definitions



s = 0;
a = 4;
i = 0;
k == 0

b = 1;

b = 2;

i < n

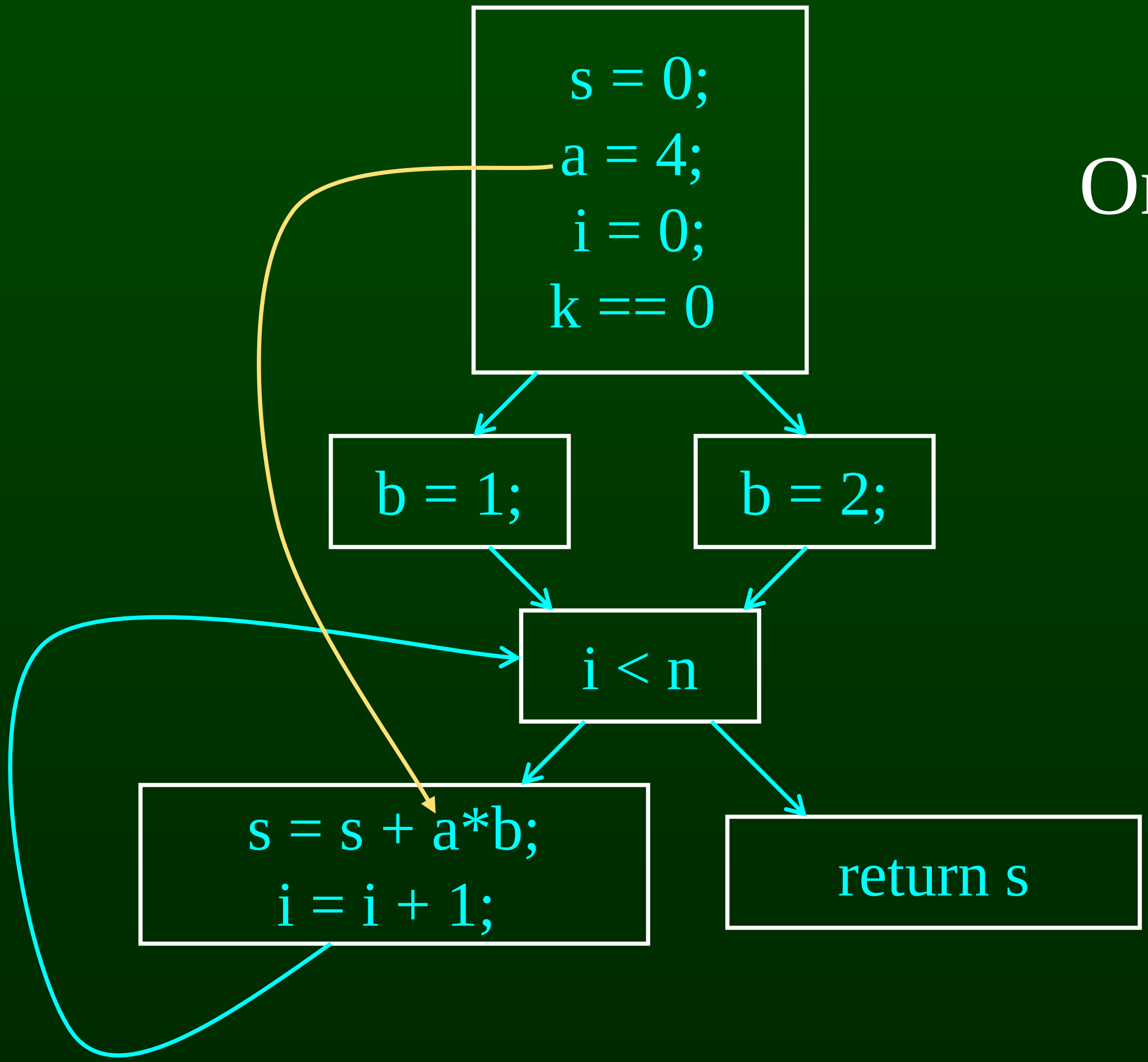
s = s + a*b;
i = i + 1;

return s

Reaching Definitions and Constant Propagation

- Is a use of a variable a constant?
    - Check all reaching definitions
    - If all assign variable to same constant
    - Then use is in fact a constant
- Can replace variable with constant

Is a Constant in s = s+a*b?



s = 0;
a = 4;
i = 0;
k == 0

b = 1;

b = 2;

i < n

s = s + a*b;
i = i + 1;

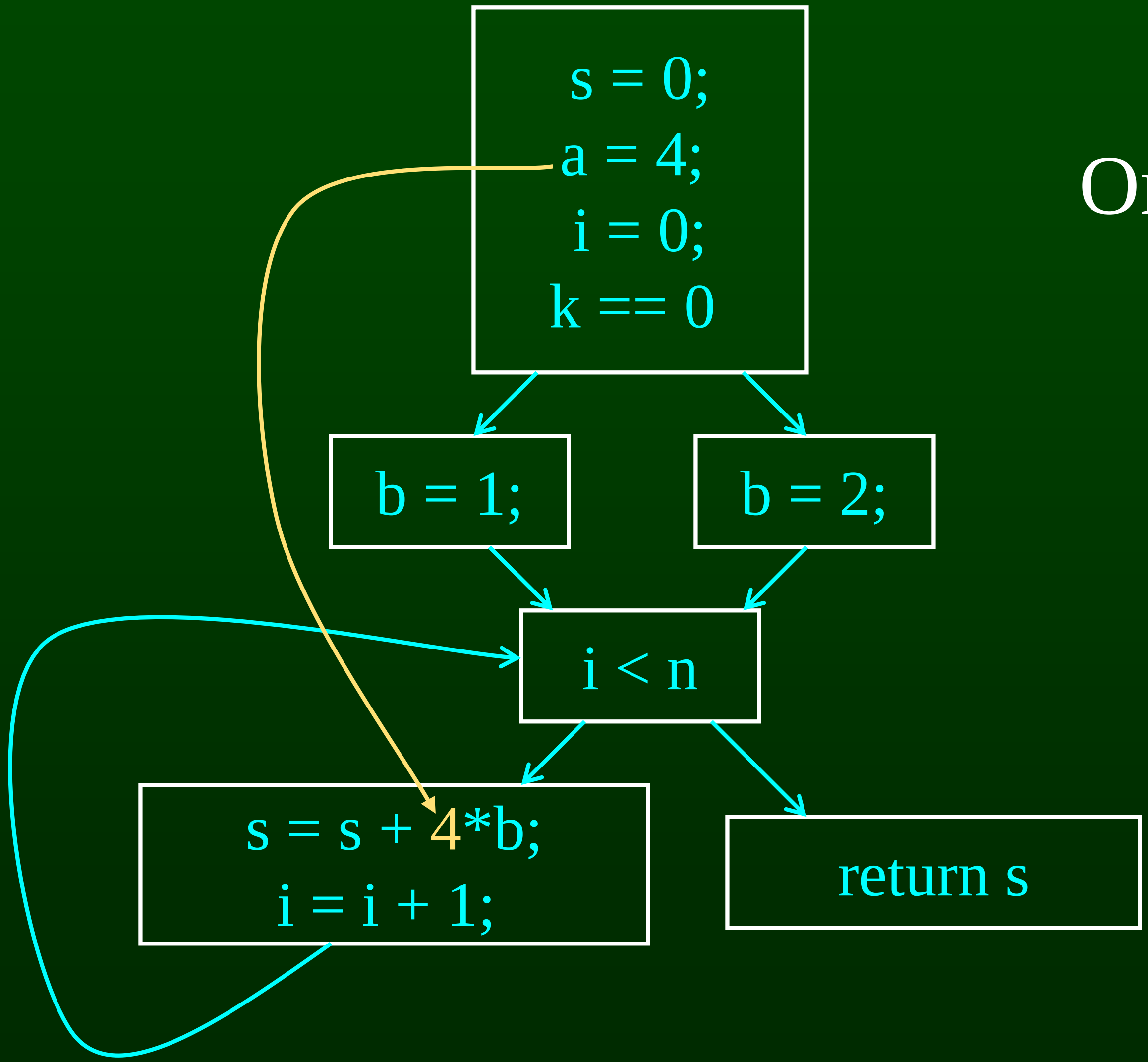
return s

Yes!

On all reaching definitions

a = 4

Constant Propagation Transform

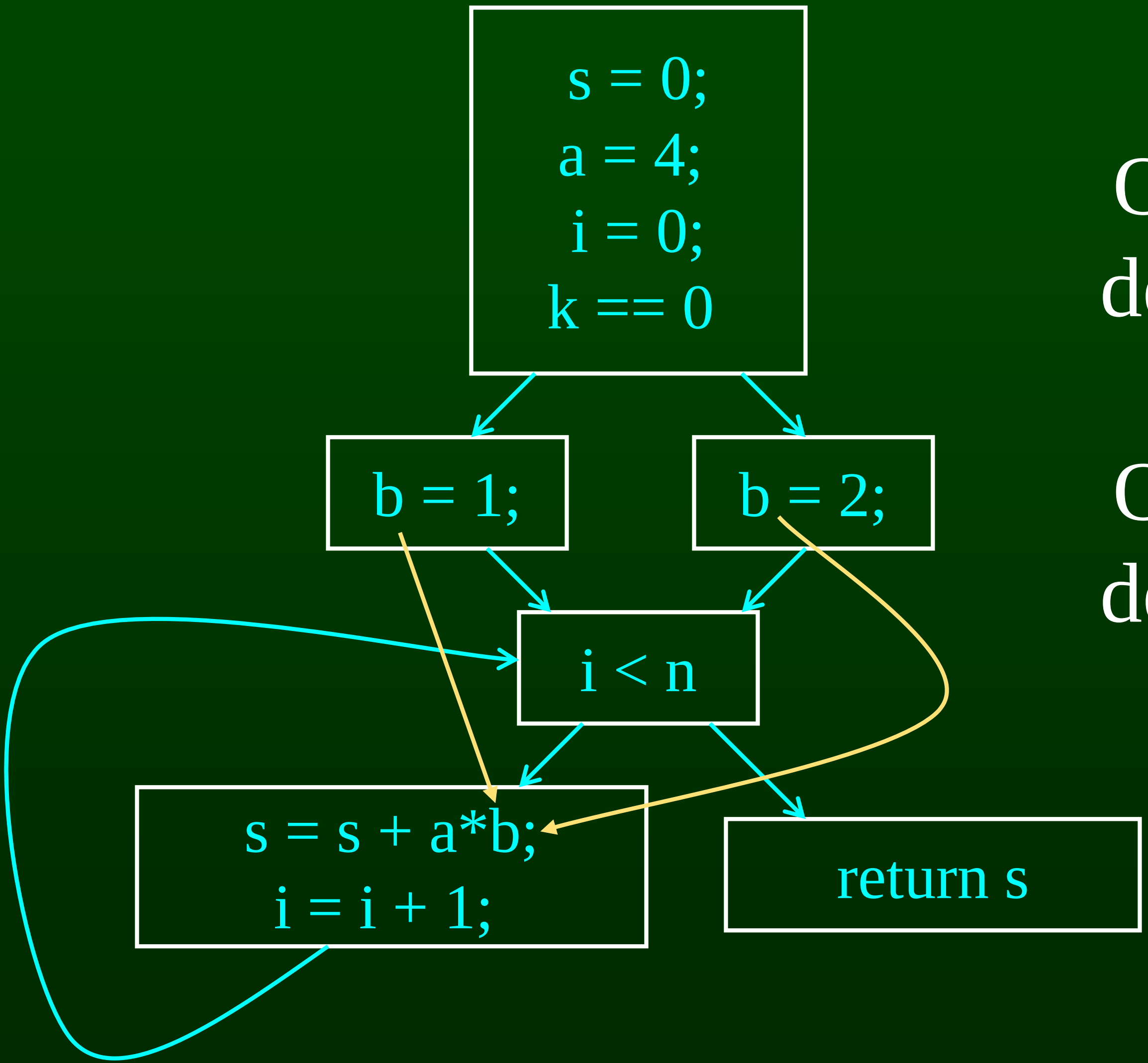


Yes!

On all reaching definitions

a = 4

Is b Constant in s = s+a*b?



s = 0;
a = 4;
i = 0;
k == 0

b = 1;

b = 2;

i < n

s = s + a*b;
i = i + 1;

return s

No!

One reaching definition with

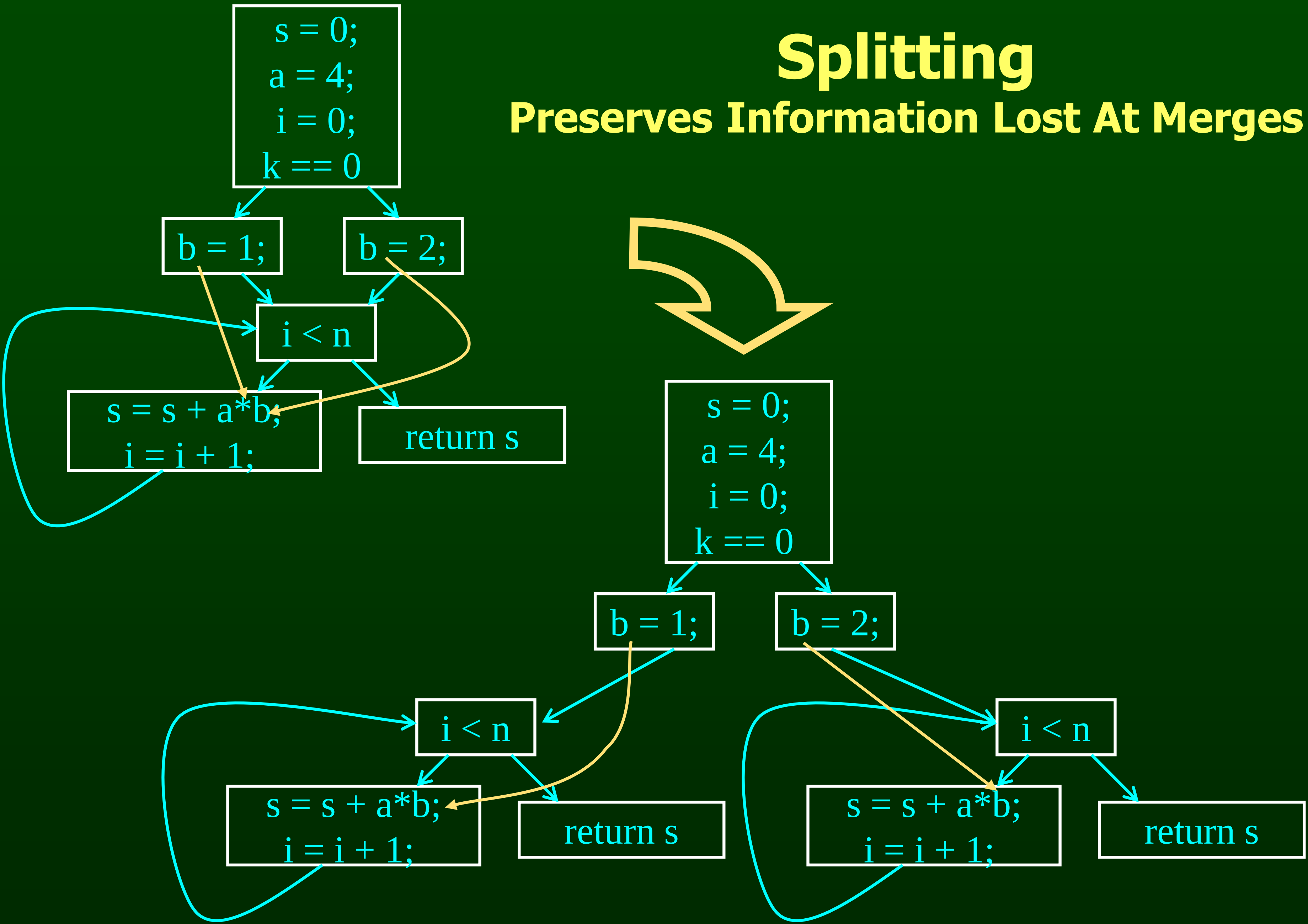
b = 1

One reaching definition with

b = 2

Splitting

Preserves Information Lost At Merges



s = 0;
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b = 1;

b = 2;

i < n

s = s + a*b;
i = i + 1;

return s

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i = 0;
k == 0

b = 1;

b = 2;

i < n

s = s + a*b;
i = i + 1;

return s

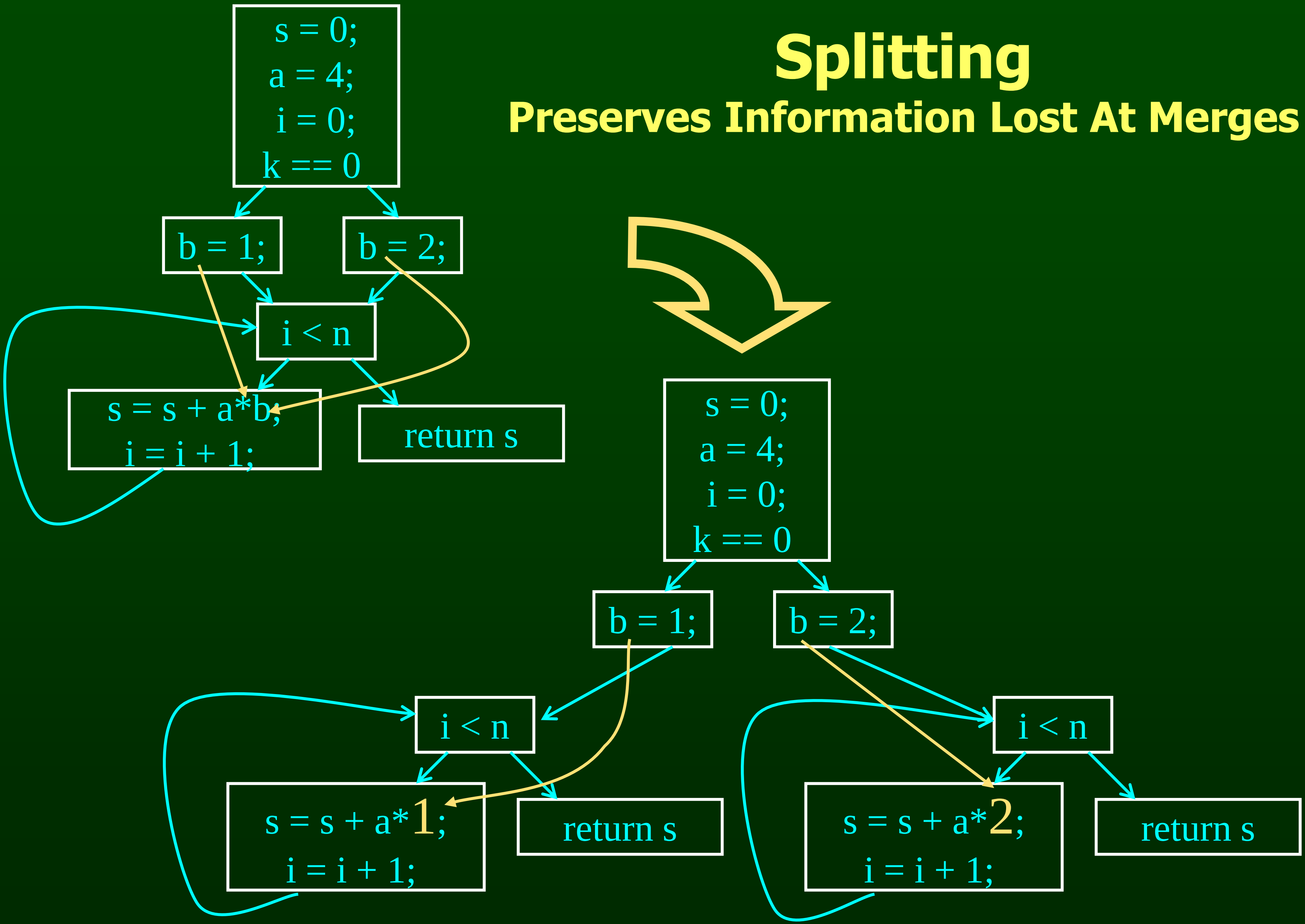
i < n

s = s + a*b;
i = i + 1;

return s

Splitting

Preserves Information Lost At Merges



s = 0;
a = 4;
i = 0;
k == 0

b = 1;

b = 2;

i < n

s = s + a*b;
i = i + 1;

return s

s = 0;
a = 4;
i = 0;
k == 0

b = 1;

b = 2;

i < n

s = s + a*1;
i = i + 1;

return s

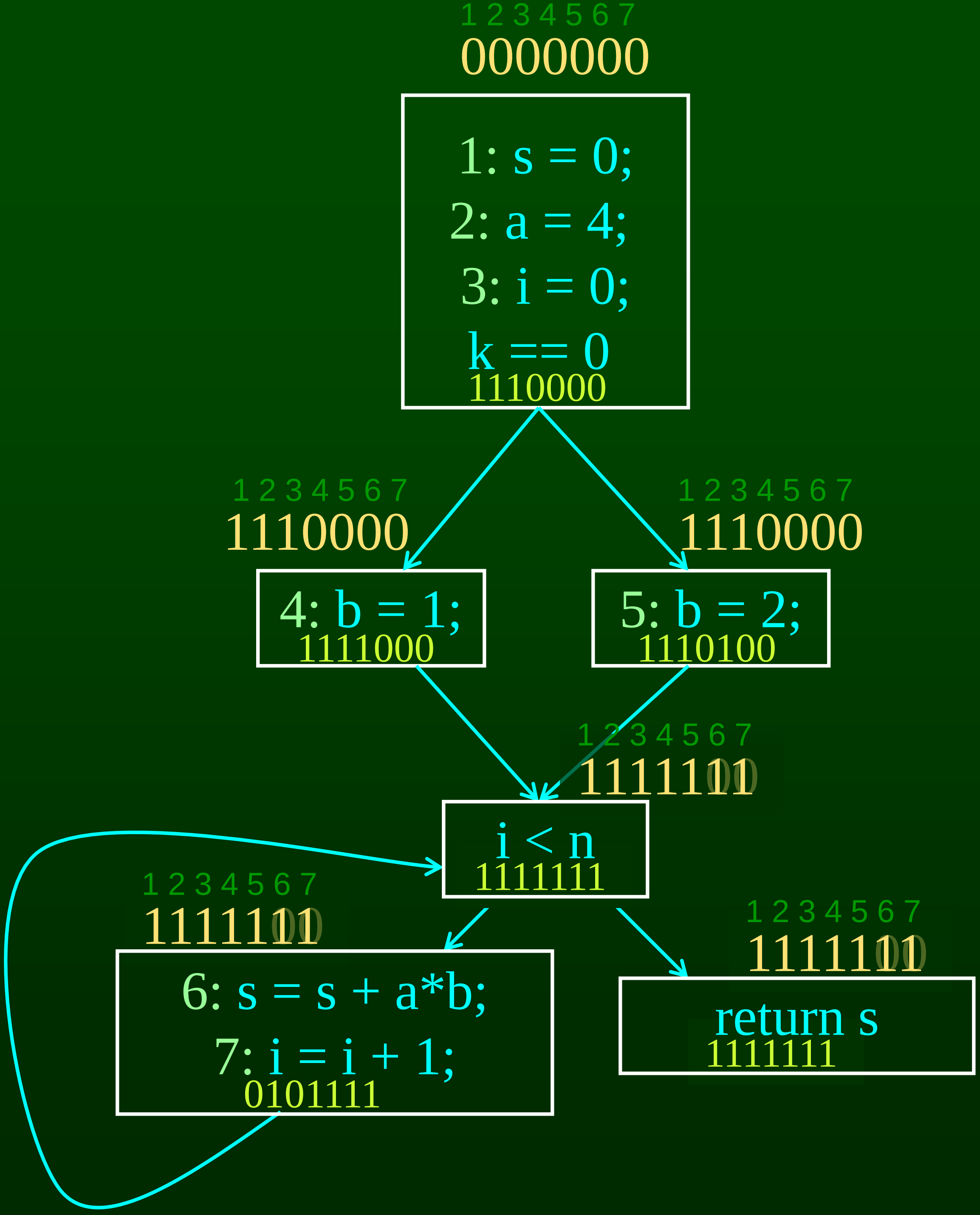
i < n

s = s + a*2;
i = i + 1;

return s

Computing Reaching Definitions

- Compute with sets of definitions
  - represent sets using bit vectors
  - each definition has a position in bit vector
- At each basic block, compute
  - definitions that reach start of block
  - definitions that reach end of block
- Do computation by simulating execution of program until reach fixed point



1 2 3 4 5 6 7
0000000

1: s = 0;
2: a = 4;
3: i = 0;
k == 0
1110000

1 2 3 4 5 6 7
1110000

4: b = 1;
1111000

1 2 3 4 5 6 7
1110000

5: b = 2;
1110100

1 2 3 4 5 6 7
1111111

i < n
1111111

1 2 3 4 5 6 7
1111111

6: s = s + a*b;
7: i = i + 1;
0101111

1 2 3 4 5 6 7
1111111

return s
1111111

Formalizing Analysis

- Each basic block has
  - IN - set of definitions that reach beginning of block
  - OUT - set of definitions that reach end of block
  - GEN - set of definitions generated in block
  - KILL - set of definitions killed in block
- GEN[s = s + a*b; i = i + 1;] = 0000011
- KILL[s = s + a*b; i = i + 1;] = 1010000
- Compiler scans each basic block to derive GEN and KILL sets

Dataflow Equations

- IN[b] = OUT[b1] U ... U OUT[bn]
  - where b1, ..., bn are predecessors of b in CFG
- OUT[b] = (IN[b] - KILL[b]) U GEN[b]
- IN[entry] = 0000000
- Result: system of equations

Solving Equations

- Use fixed point algorithm
- Initialize with solution of OUT[b] = 0000000
- Repeatedly apply equations
  - IN[b] = OUT[b1] U ... U OUT[bn]
  - OUT[b] = (IN[b] - KILL[b]) U GEN[b]
- Until reach fixed point
- Until equation application has no further effect
- Use a worklist to track which equation applications may have a further effect

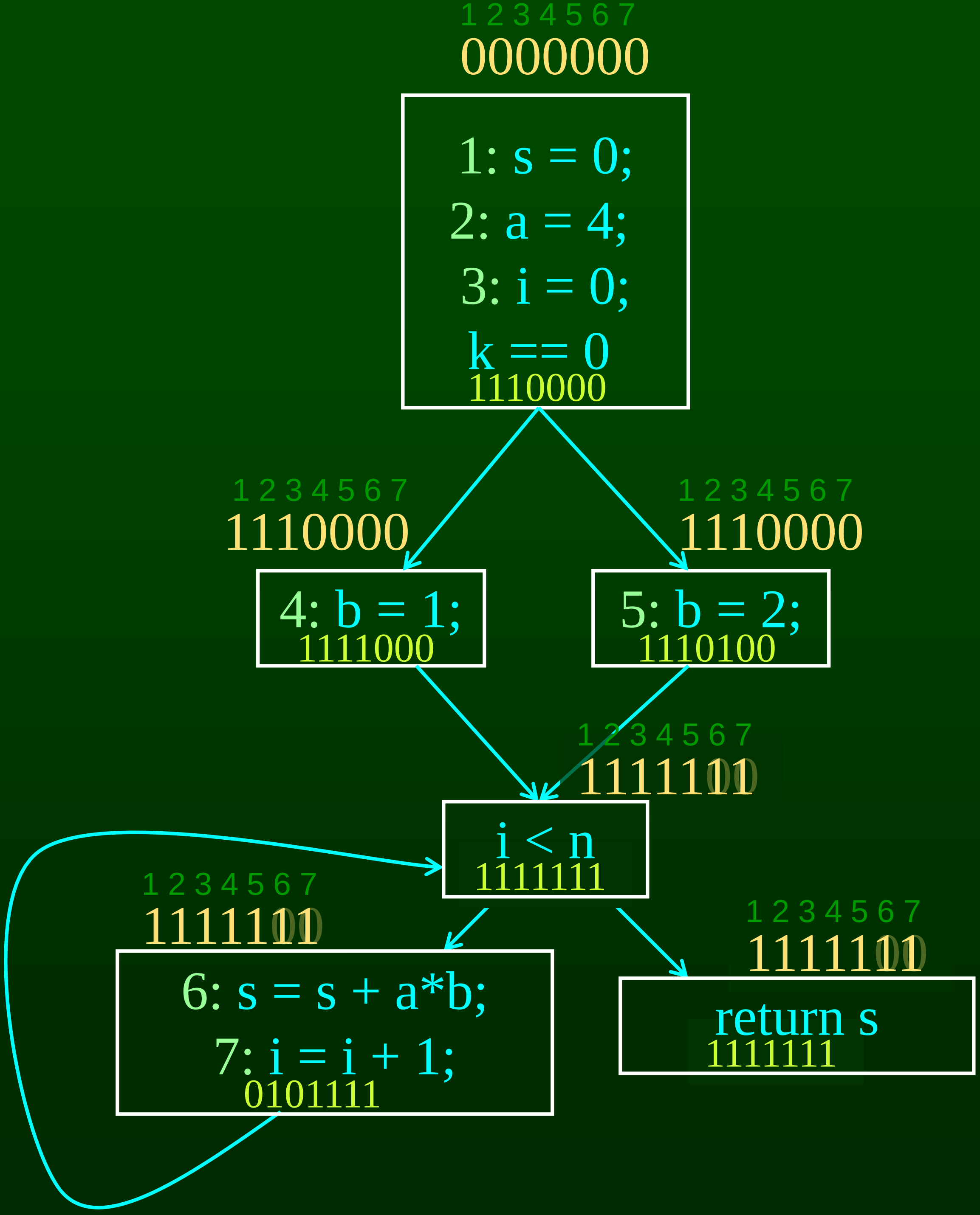
Reaching Definitions Algorithm

```
for all nodes n in N
    OUT[n] = emptyset; // OUT[n] = GEN[n];
IN[Entry] = emptyset;
OUT[Entry] = GEN[Entry];
Changed = N - { Entry }; // N = all nodes in graph

while (Changed != emptyset)
    choose a node n in Changed;
    Changed = Changed - { n };

    IN[n] = emptyset;
    for all nodes p in predecessors(n)
        IN[n] = IN[n] U OUT[p];

    OUT[n] = GEN[n] U (IN[n] - KILL[n]);

    if (OUT[n] changed)
        for all nodes s in successors(n)
            Changed = Changed U { s };
```

Questions

- Does the algorithm halt?
  - yes, because transfer function is monotonic
  - if increase IN, increase OUT
  - in limit, all bits are 1

- If bit is 0, does the corresponding definition ever reach basic block?
- If bit is 1, is does the corresponding definition always reach the basic block?



1 2 3 4 5 6 7
0000000

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3: i = 0;
k == 0
1110000

1 2 3 4 5 6 7
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1 2 3 4 5 6 7
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i < n
1111111

1 2 3 4 5 6 7
1111111

6: s = s + a*b;
7: i = i + 1;
0101111

1 2 3 4 5 6 7
1111111

return s
1111111

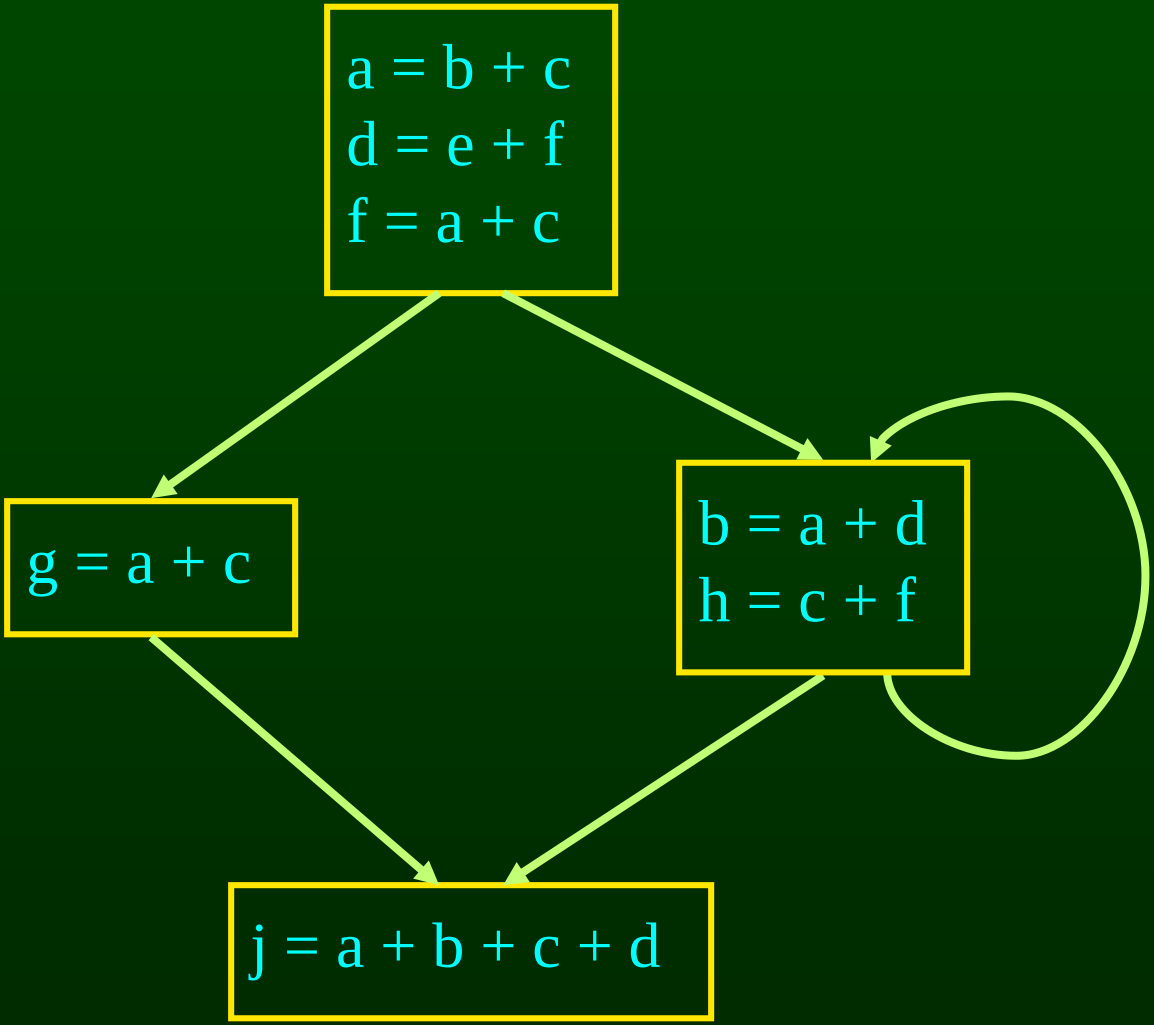
Outline

- Reaching Definitions
- **Available Expressions**
- Liveness

Available Expressions

- An expression x+y is available at a point p if
  - every path from the initial node to p must evaluate x+y before reaching p,
  - and there are no assignments to x or y after the evaluation but before p.
- Available Expression information can be used to do global (across basic blocks) CSE
- If expression is available at use, no need to reevaluate it

Example: Available Expression



a = b + c
d = e + f
f = a + c

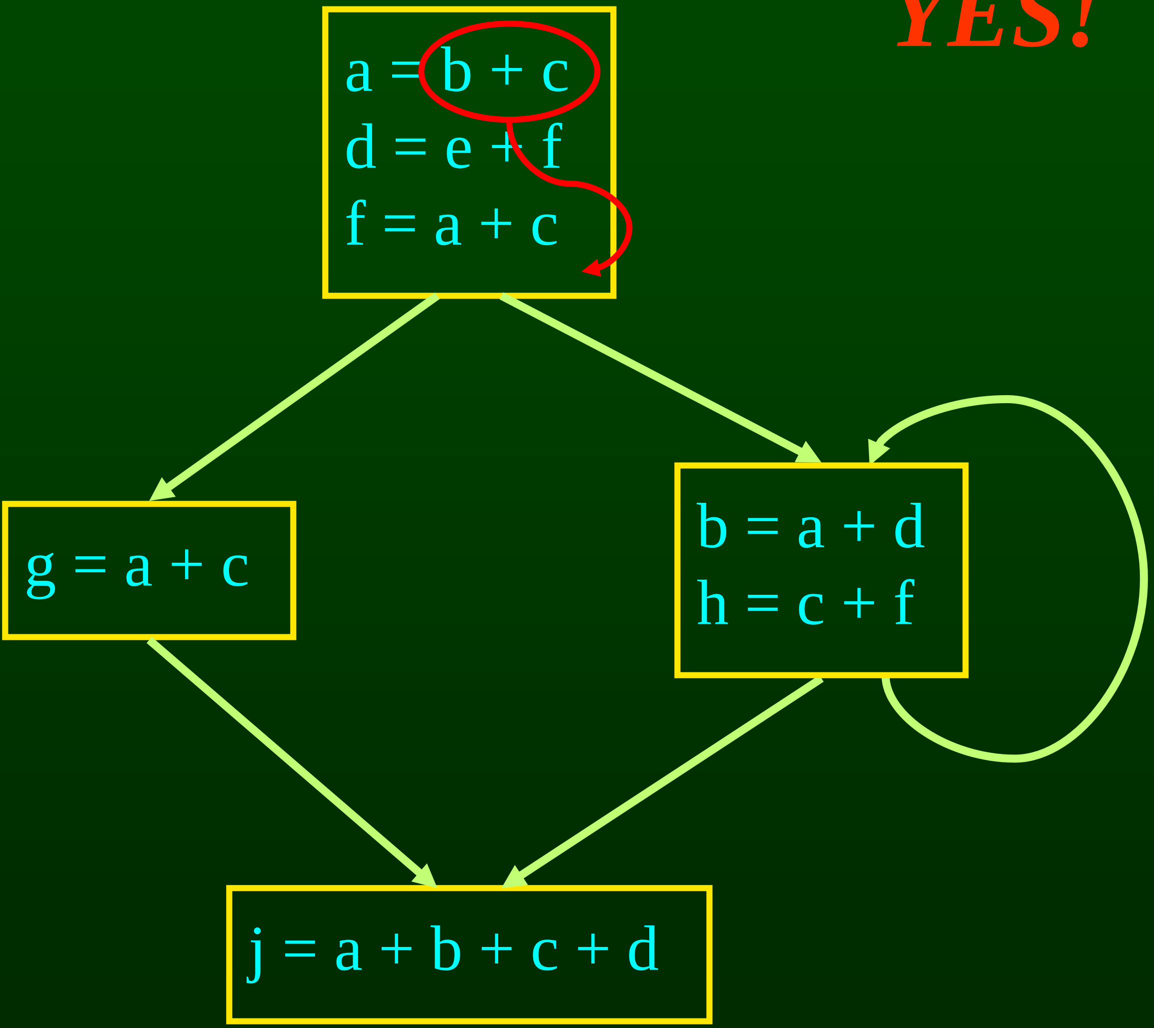
g = a + c

b = a + d
h = c + f

j = a + b + c + d

Is the Expression Available?

YES!



a = b + c
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f = a + c

g = a + c

b = a + d
h = c + f

j = a + b + c + d

Is the Expression Available?

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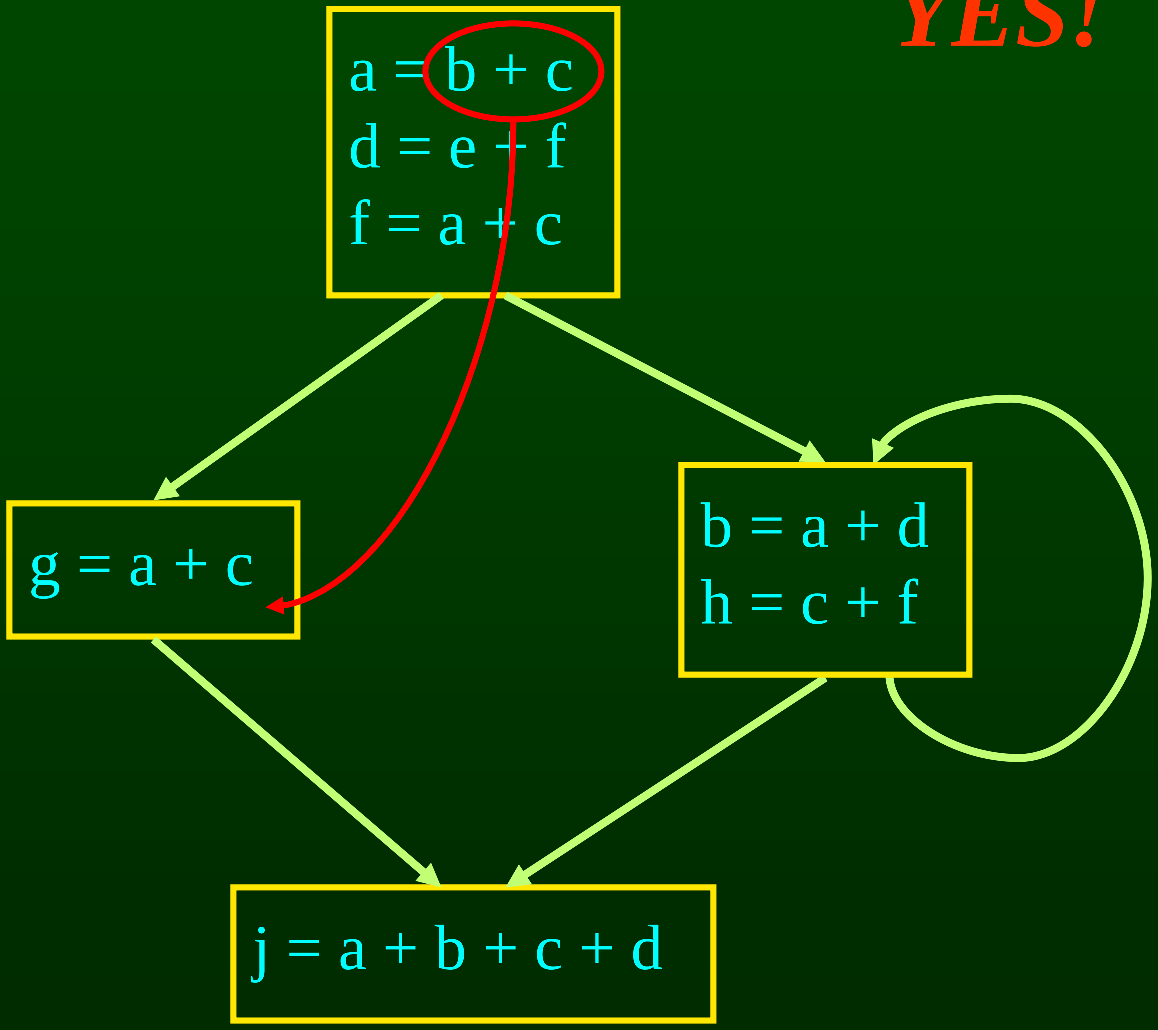

a = b + c
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f = a + c

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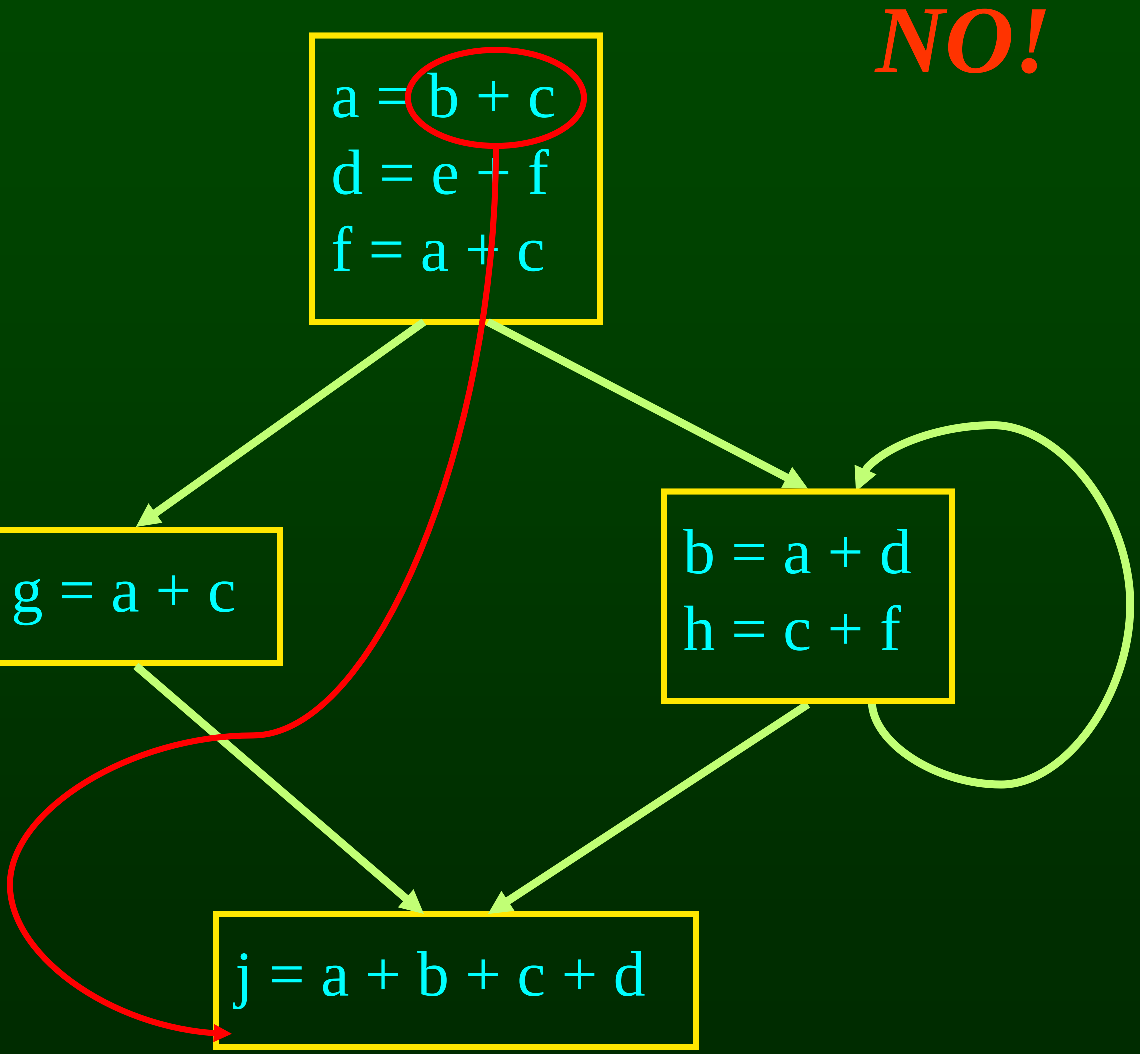
b = a + d
h = c + f

j = a + b + c + d

Is the Expression Available?



NO!

a = b + c
d = e + f
f = a + c

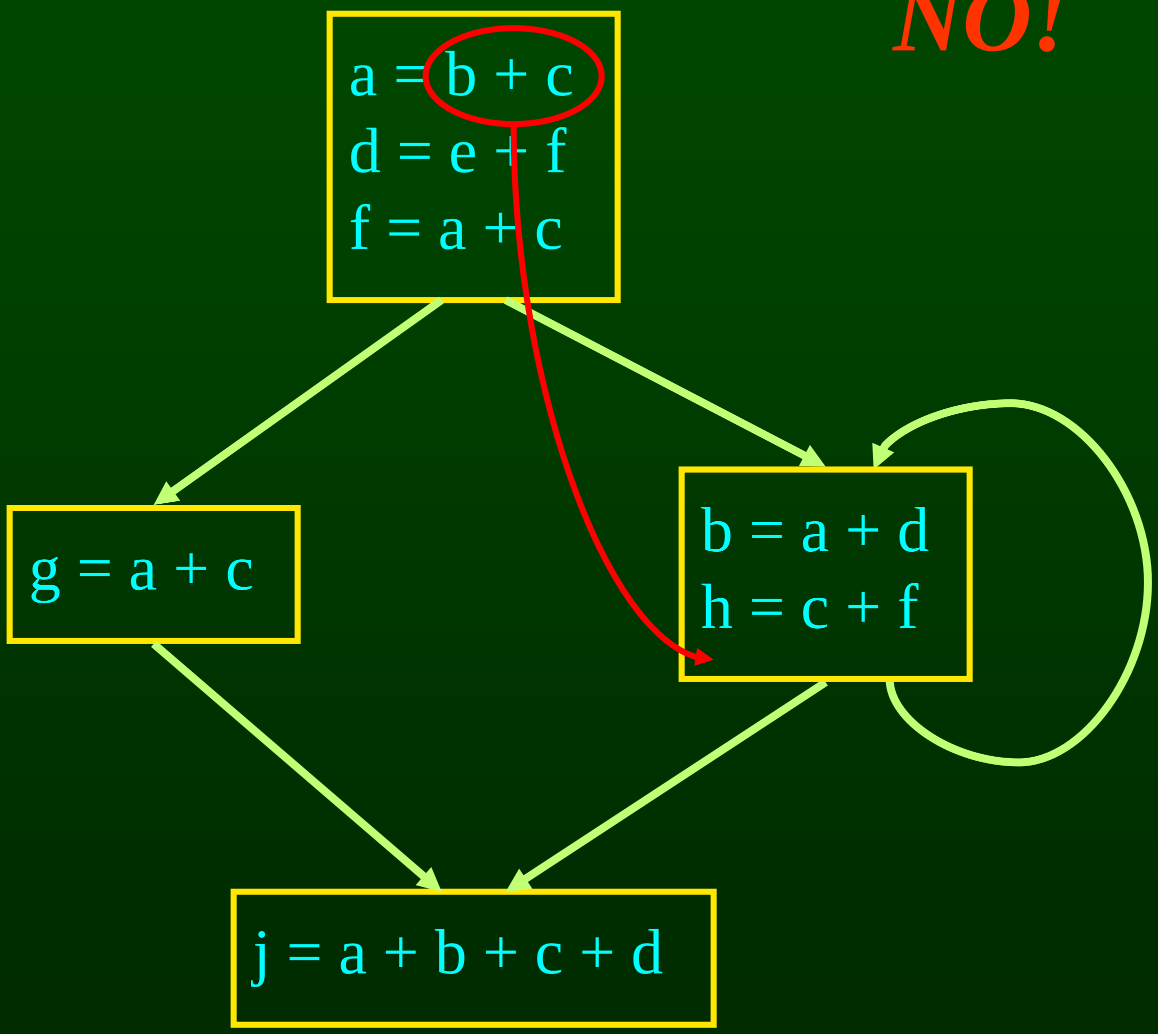
g = a + c

b = a + d
h = c + f

j = a + b + c + d

Is the Expression Available?

NO!



a = b + c
d = e + f
f = a + c

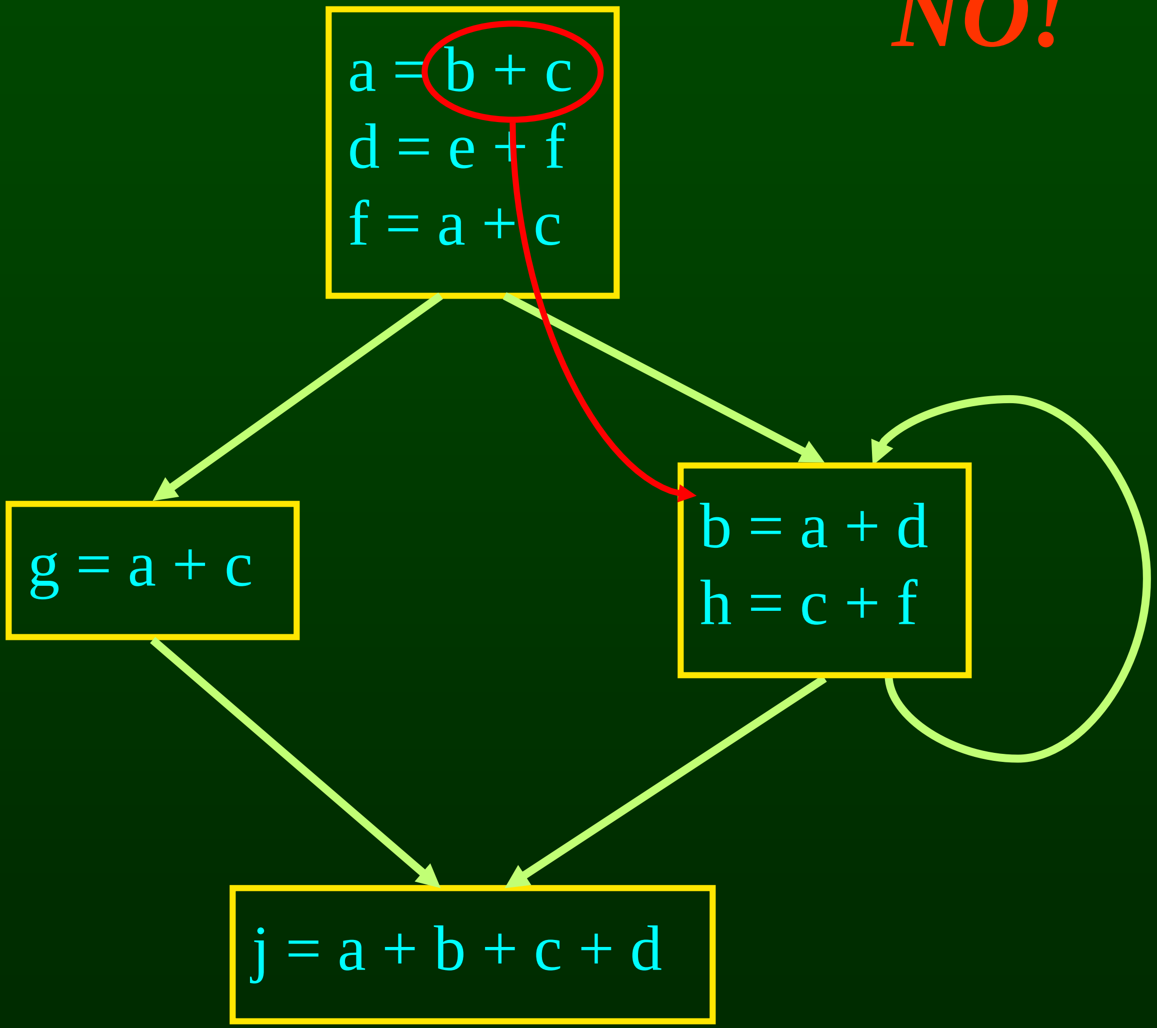
g = a + c

b = a + d
h = c + f

j = a + b + c + d

Is the Expression Available?

NO!



a = b + c
d = e + f
f = a + c

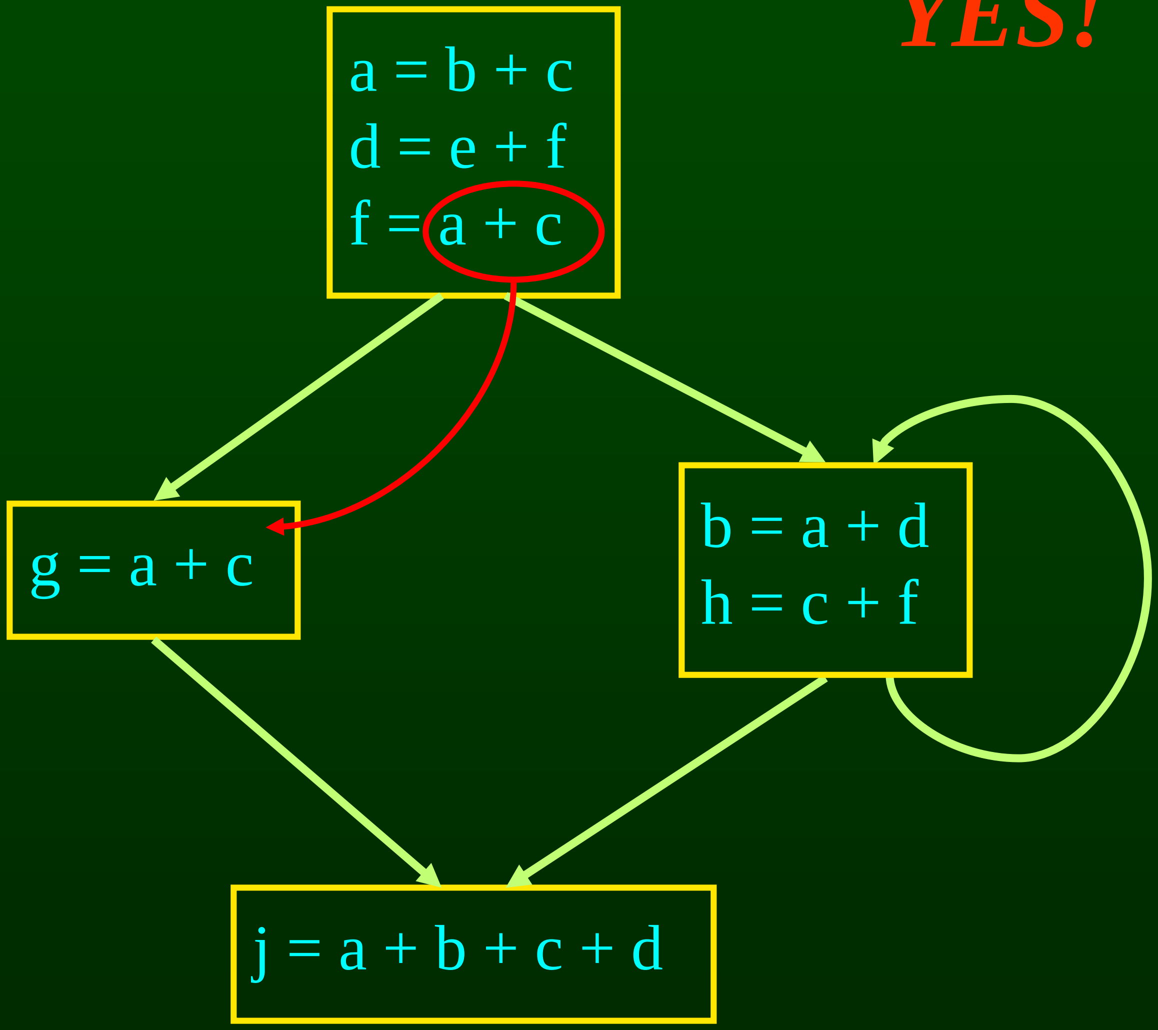
g = a + c

b = a + d
h = c + f

j = a + b + c + d

Is the Expression Available?

YES!



a = b + c
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f = a + c

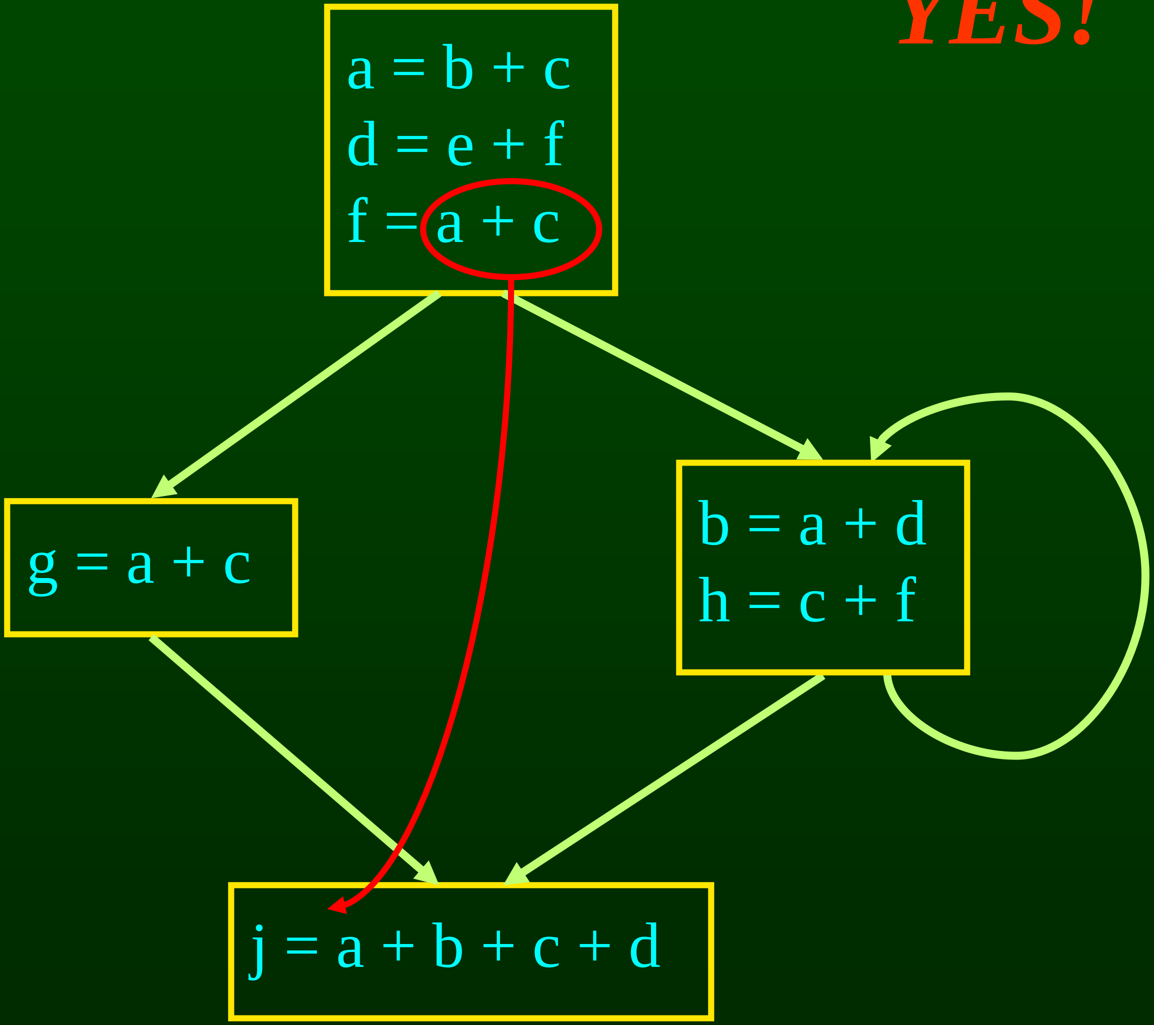
g = a + c

b = a + d
h = c + f

j = a + b + c + d

Is the Expression Available?

YES!



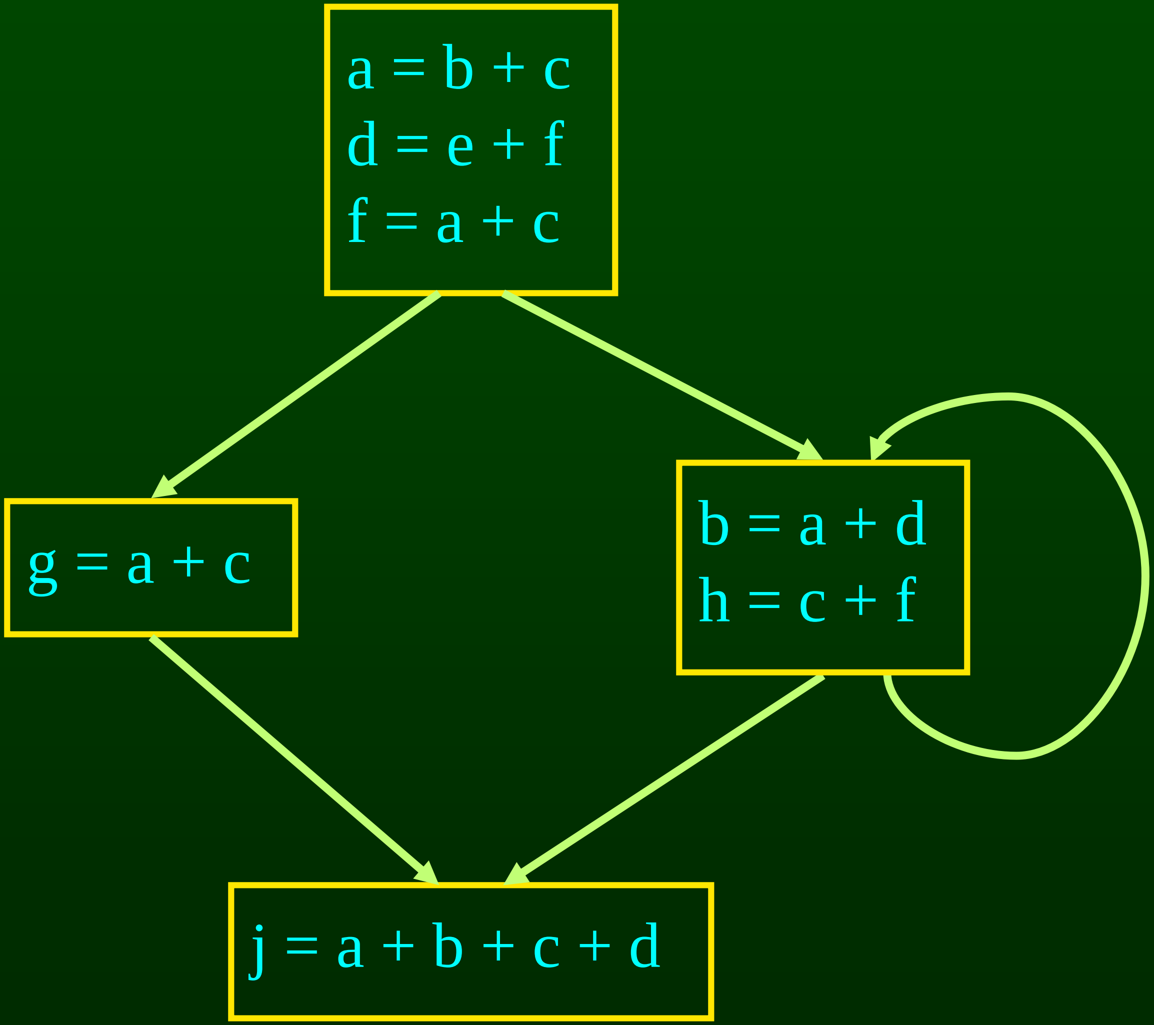
a = b + c
d = e + f
f = a + c

g = a + c

b = a + d
h = c + f

j = a + b + c + d

Use of Available Expressions



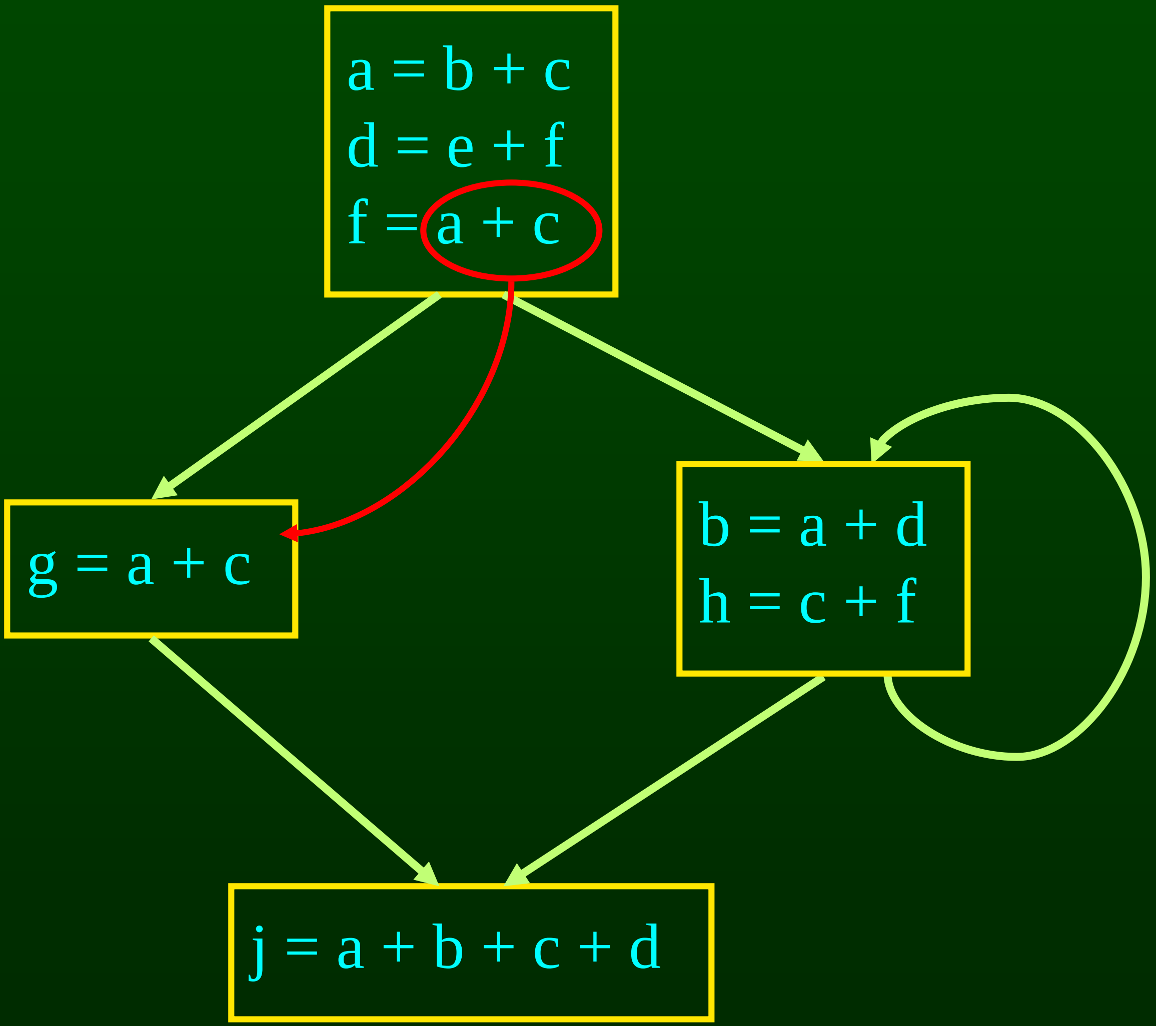
a = b + c
d = e + f
f = a + c

g = a + c

b = a + d
h = c + f

j = a + b + c + d

Use of Available Expressions



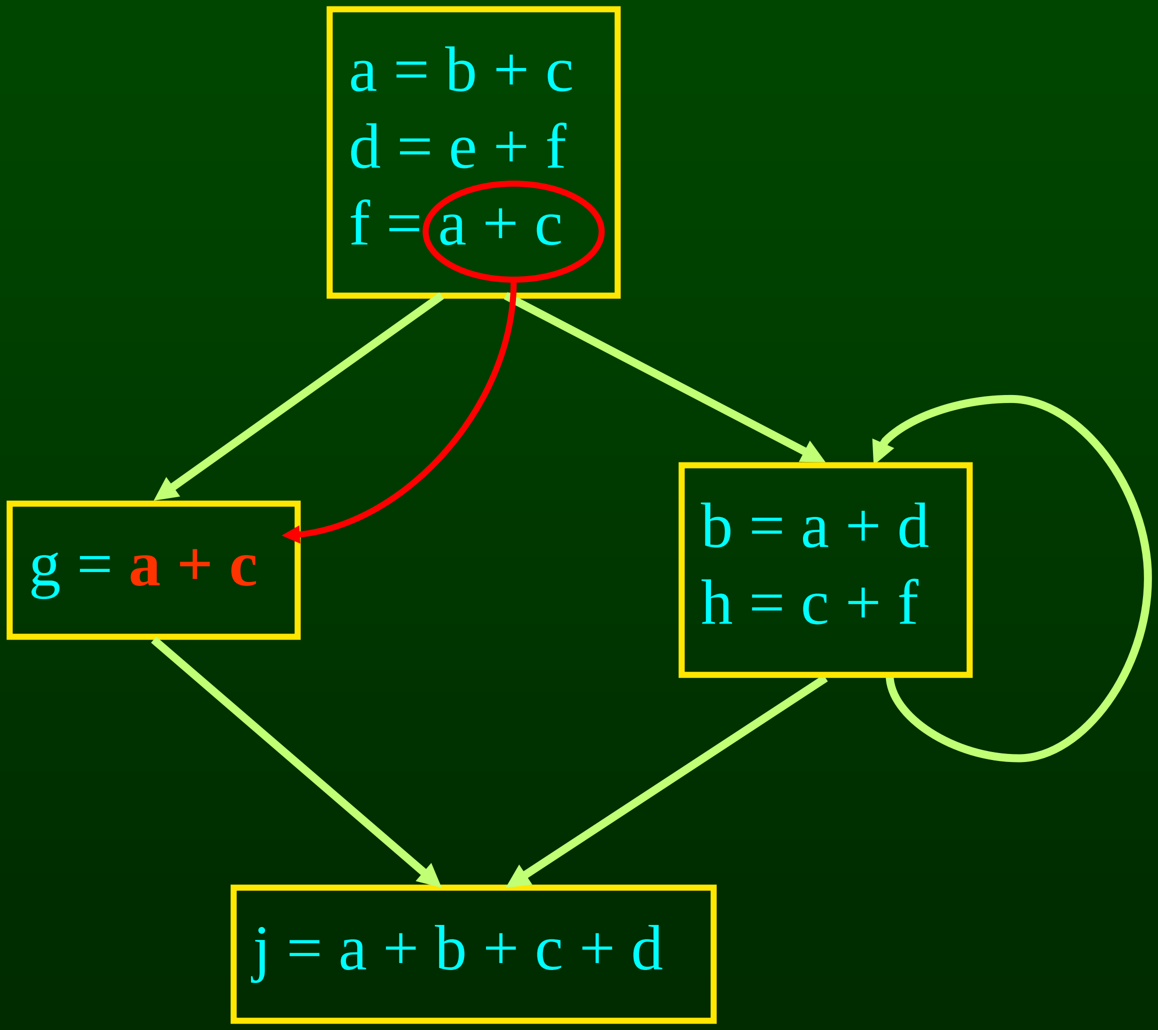
a = b + c
d = e + f
f = a + c

g = a + c

b = a + d
h = c + f

j = a + b + c + d

Use of Available Expressions



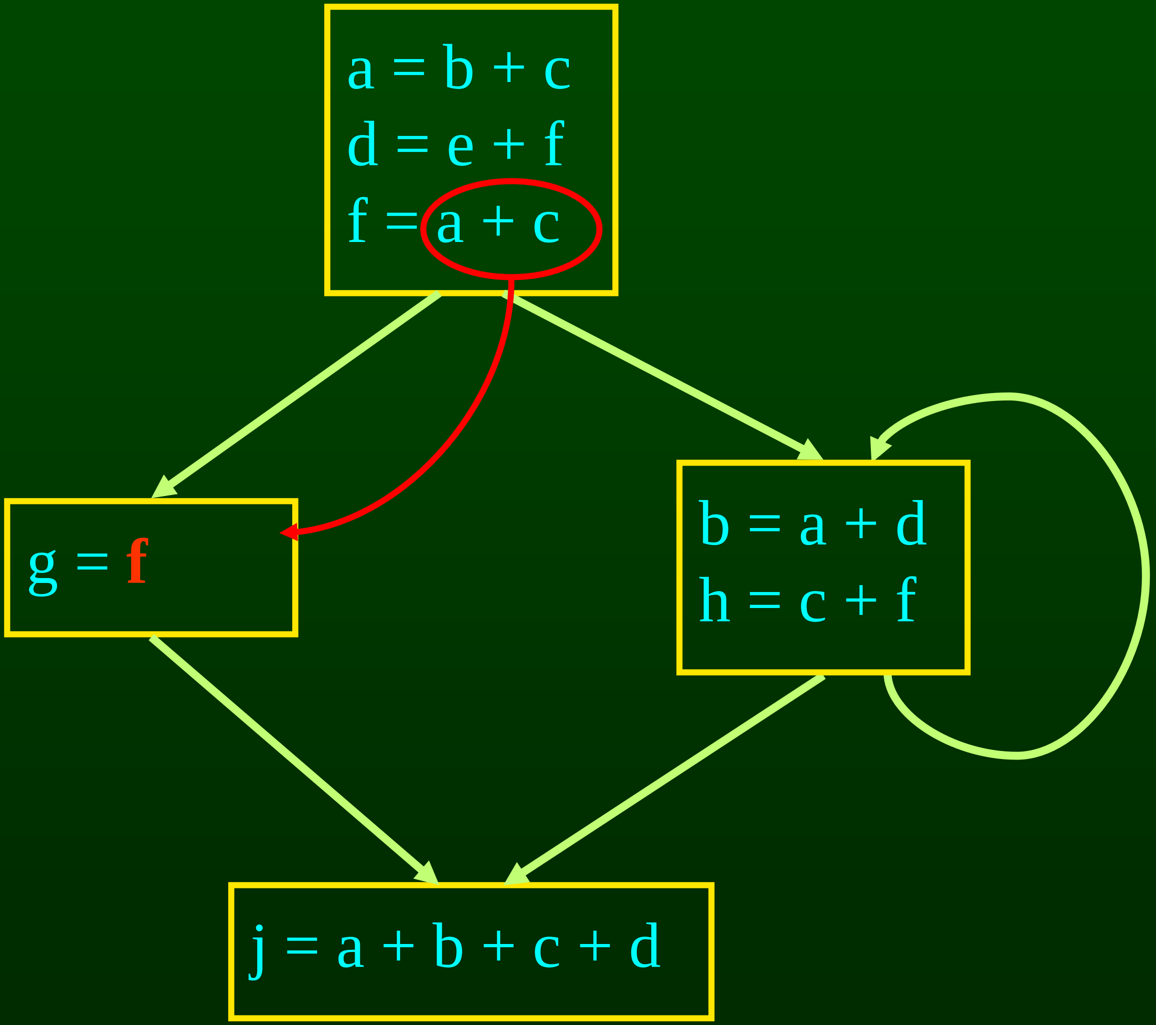
a = b + c
d = e + f
f = a + c

g = **a + c**

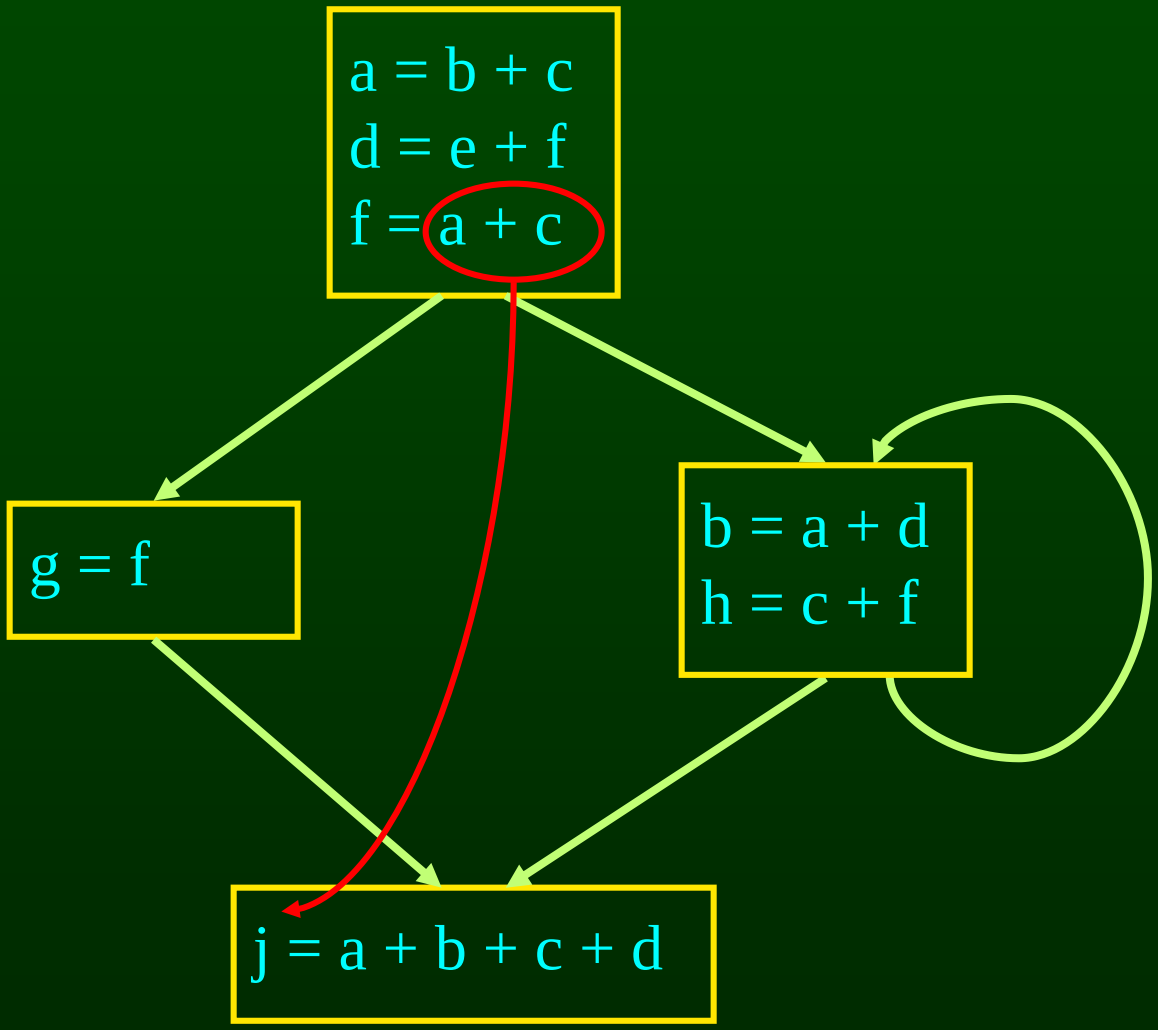
b = a + d
h = c + f

j = a + b + c + d

Use of Available Expressions



Use of Available Expressions



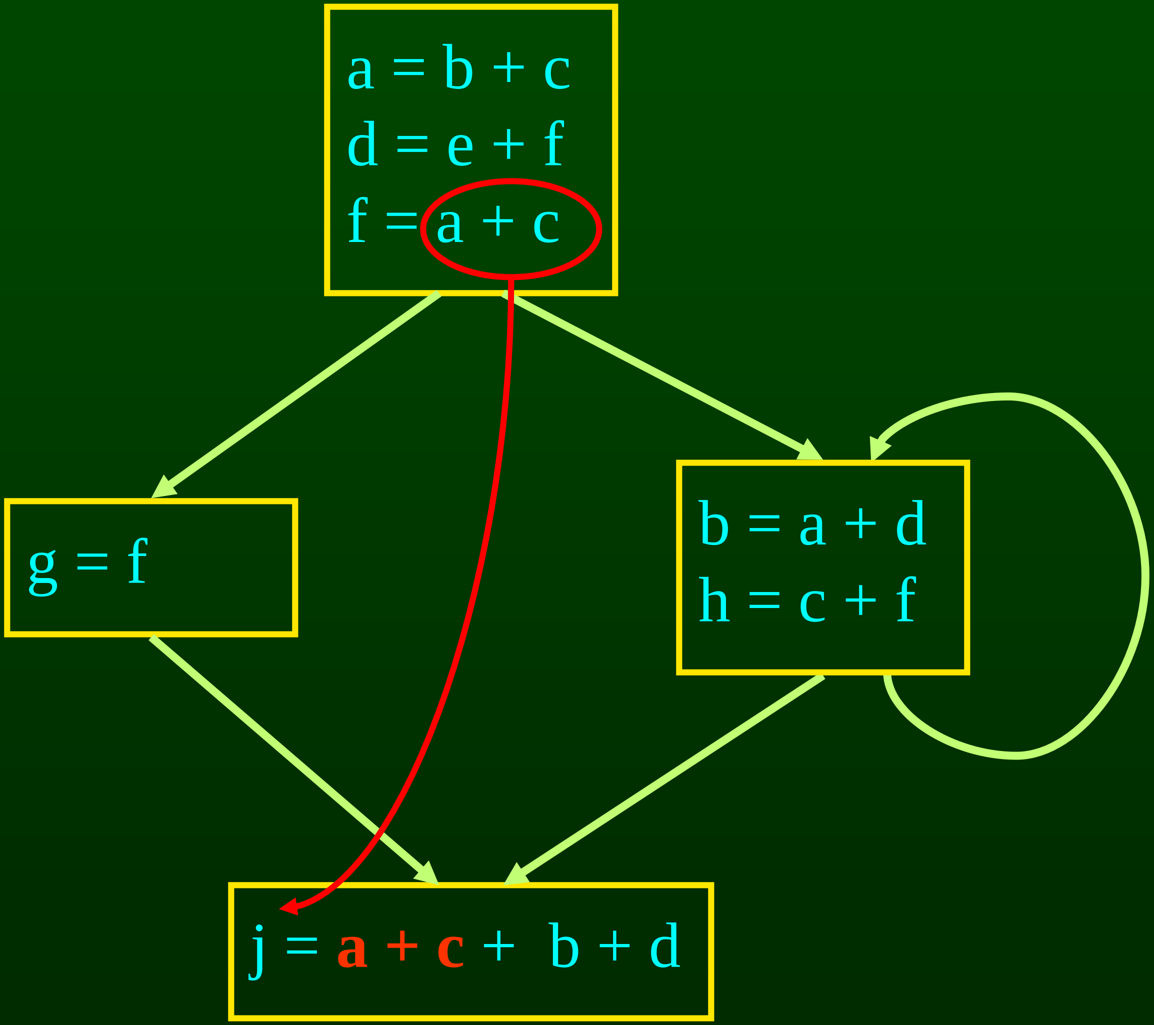
a = b + c
d = e + f
f = a + c

g = f

b = a + d
h = c + f

j = a + b + c + d

Use of Available Expressions



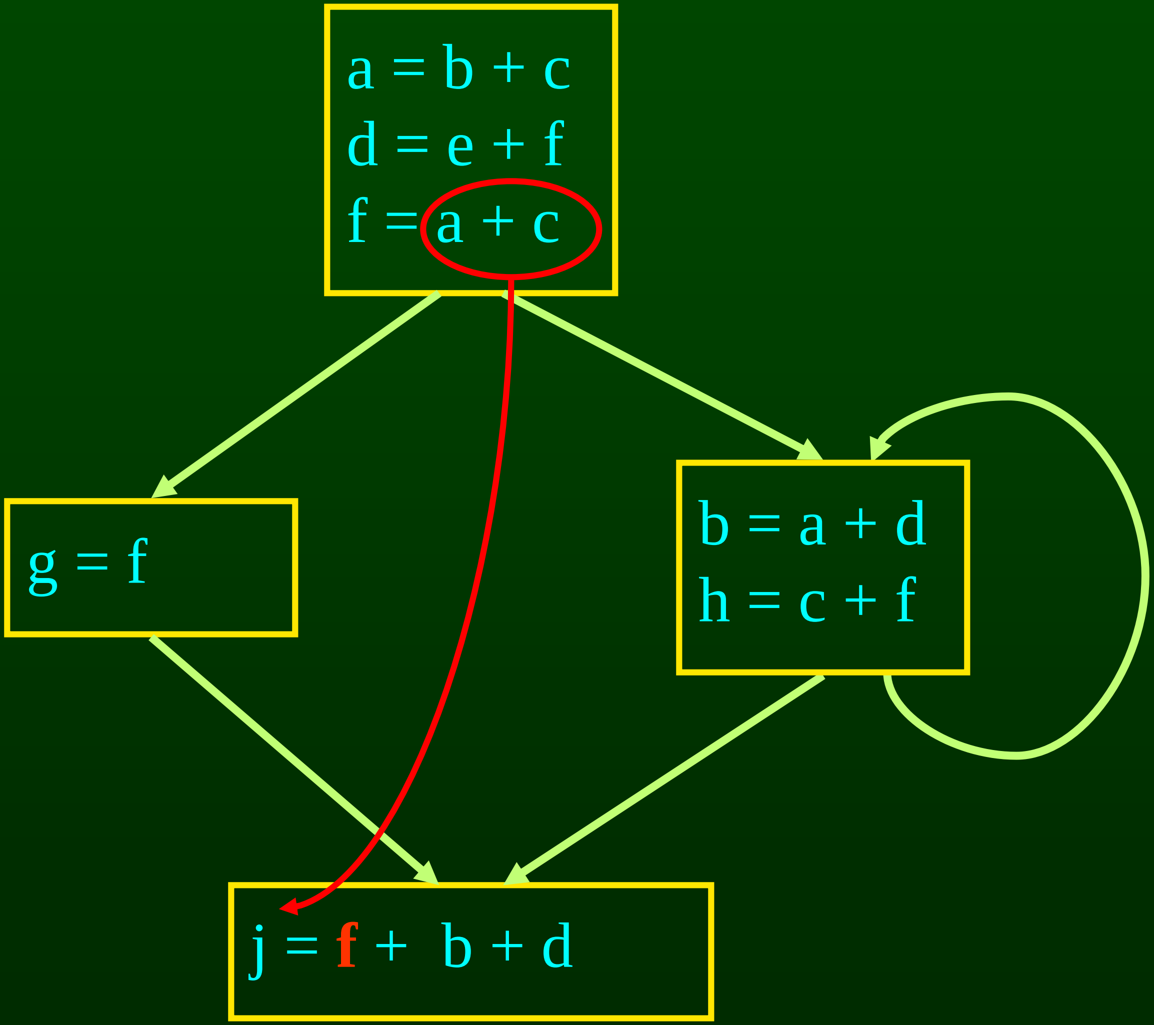
a = b + c
d = e + f
f = a + c

g = f

b = a + d
h = c + f

j = **a + c** + b + d

Use of Available Expressions



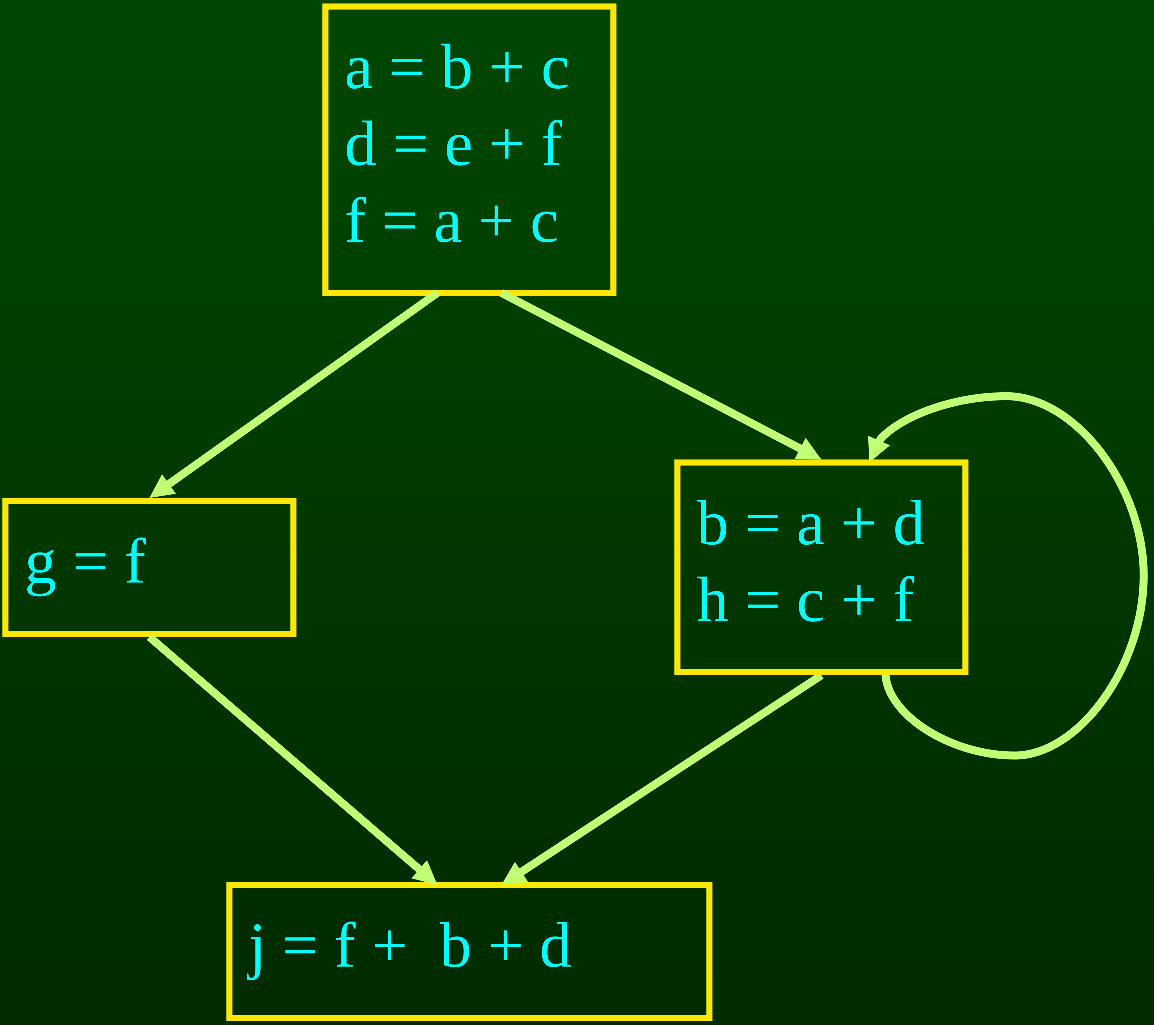
a = b + c
d = e + f
f = a + c

g = f

b = a + d
h = c + f

j = **f** + b + d

Use of Available Expressions



a = b + c
d = e + f
f = a + c

g = f

b = a + d
h = c + f

j = f + b + d

Computing Available Expressions

- Represent sets of expressions using bit vectors
- Each expression corresponds to a bit
- Run dataflow algorithm similar to reaching definitions
- Big difference
  - definition reaches a basic block if it comes from ANY predecessor in CFG
  - expression is available at a basic block only if it is available from ALL predecessors in CFG

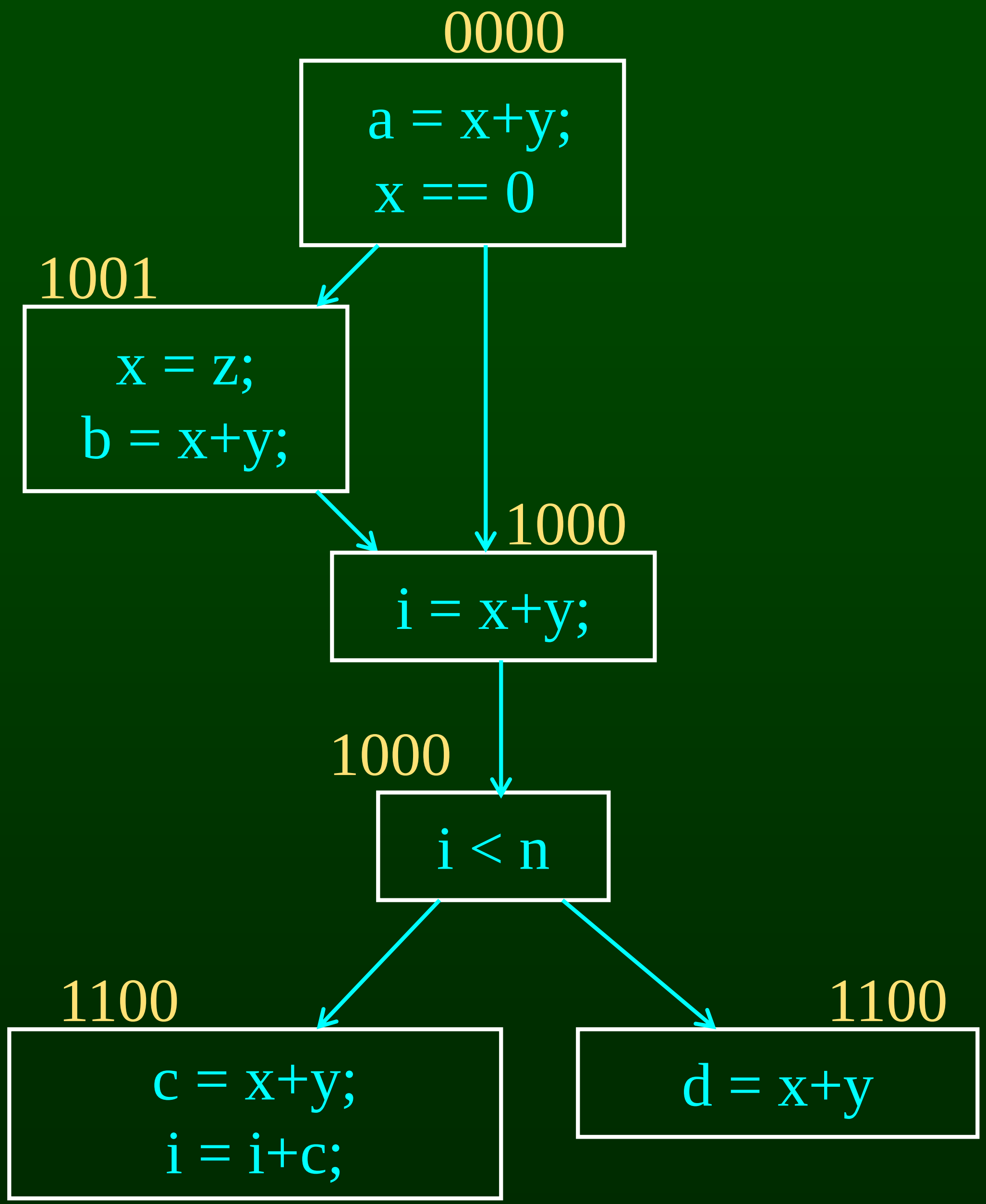
Expressions

1: x+y

2: i<n

3: i+c

4: x==0



0000

a = x+y;

x == 0

1001

x = z;

b = x+y;

1000

i = x+y;

1000

i < n

1100

c = x+y;

i = i+c;

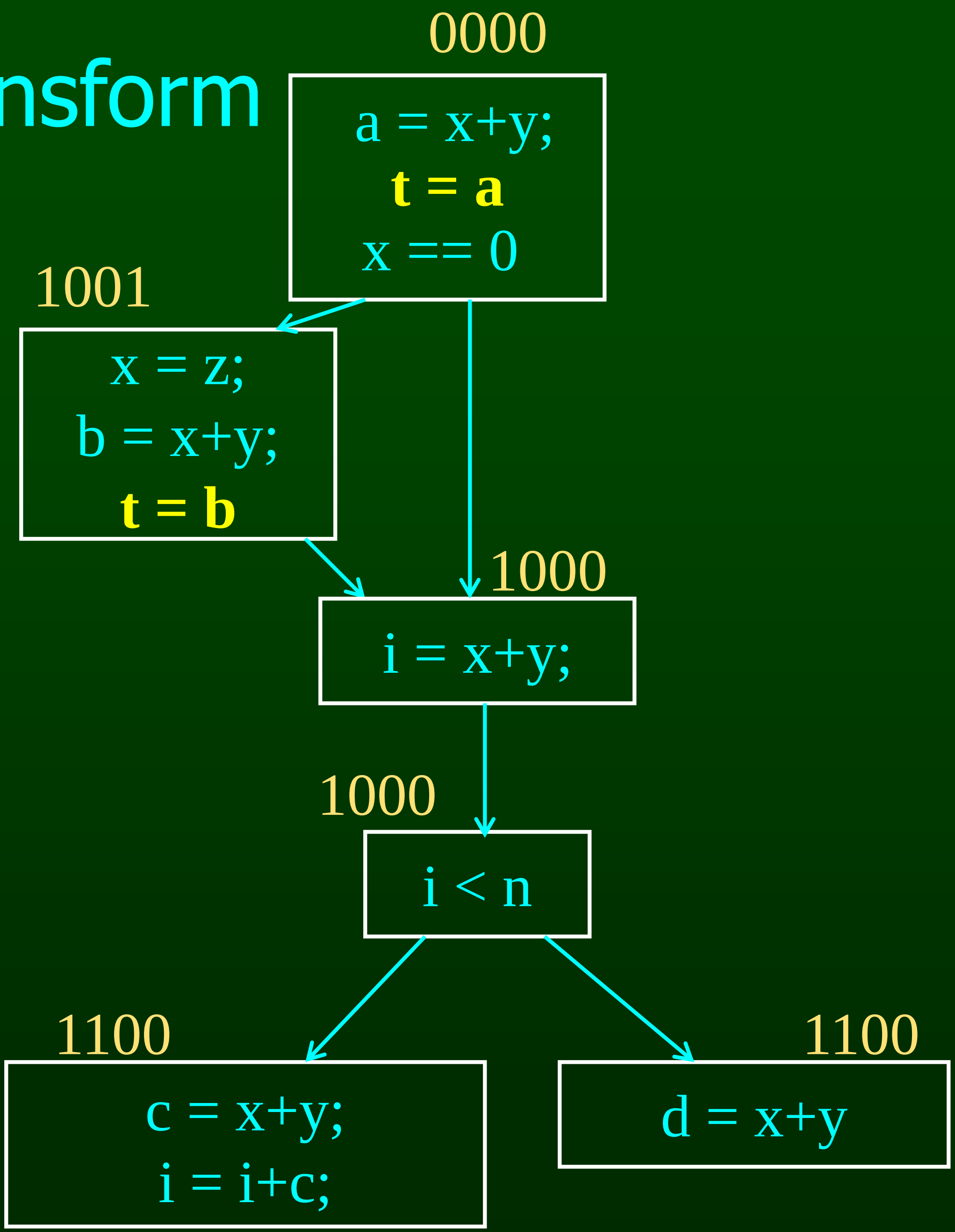
1100

d = x+y

Global CSE Transform

Expressions
1: x+y
2: i<n
3: i+c
4: x==0

must use same temp
for CSE in all blocks



0000
a = x+y;
t = a
x == 0

1001
x = z;
b = x+y;
t = b

1000
i = x+y;

1000
i < n

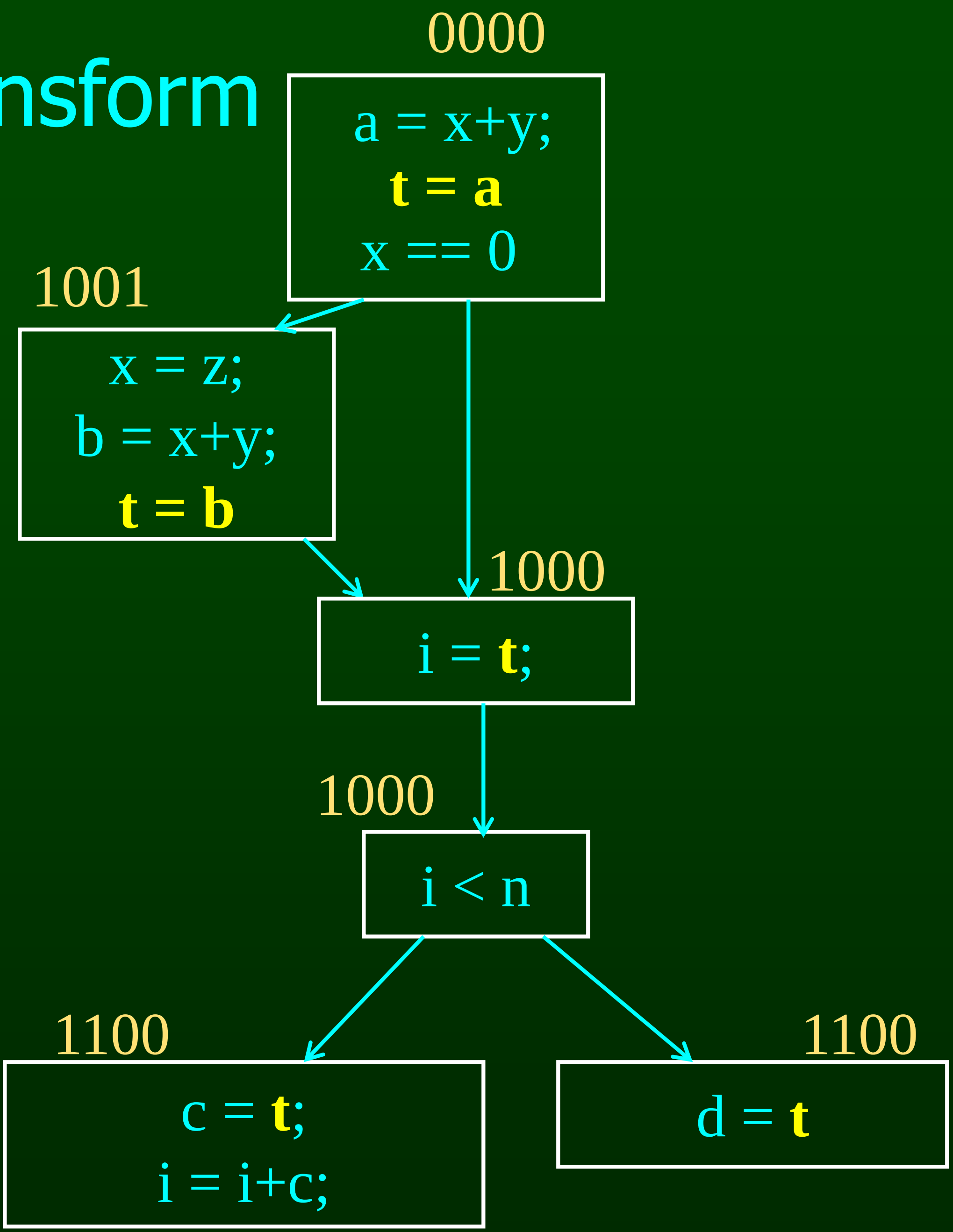
1100
c = x+y;
i = i+c;

1100
d = x+y

Global CSE Transform

Expressions
1: x+y
2: i<n
3: i+c
4: x==0

must use same temp
for CSE in all blocks



Formalizing Analysis

- Each basic block has
  - IN - set of expressions available at start of block
  - OUT - set of expressions available at end of block
  - GEN - set of expressions computed in block
  - KILL - set of expressions killed in in block
- GEN[x = z; b = x+y] = 1000
- KILL[x = z; b = x+y] = 1001
- Compiler scans each basic block to derive GEN and KILL sets

Dataflow Equations

- $IN[b] = OUT[b1] \cap \ldots \cap OUT[bn]$
  - where b1, ..., bn are predecessors of b in CFG
- $OUT[b] = (IN[b] - KILL[b]) \cup GEN[b]$
- $IN[entry] = 0000$
- Result: system of equations

Solving Equations

- Use fixed point algorithm
- IN[entry] = 0000
- Initialize OUT[b] = 1111
- Repeatedly apply equations
  - IN[b] = OUT[b1] ∩ ... ∩ OUT[bn]
  - OUT[b] = (IN[b] - KILL[b]) U GEN[b]
- Use a worklist algorithm to reach fixed point

Available Expressions Algorithm

```
for all nodes n in N
    OUT[n] = E;  // OUT[n] = E - KILL[n];
IN[Entry] = emptyset;
OUT[Entry] = GEN[Entry];
Changed = N - { Entry }; // N = all nodes in graph

while (Changed != emptyset)
    choose a node n in Changed;
    Changed = Changed - { n };

    IN[n] = E; // E is set of all expressions
    for all nodes p in predecessors(n)
        IN[n] = IN[n] ∩ OUT[p];

    OUT[n] = GEN[n] U (IN[n] - KILL[n]);

    if (OUT[n] changed)
        for all nodes s in successors(n)
            Changed = Changed U { s };
```

Questions

- Does algorithm always halt?
- If expression is available in some execution, is it always marked as available in analysis?
- If expression is not available in some execution, can it be marked as available in analysis?

Duality In Two Algorithms

- Reaching definitions
  - Confluence operation is set union
  - OUT[b] initialized to empty set
- Available expressions
  - Confluence operation is set intersection
  - OUT[b] initialized to set of available expressions
- General framework for dataflow algorithms.
- Build parameterized dataflow analyzer once, use for all dataflow problems

Outline

- Reaching Definitions
- Available Expressions
- **Liveness**

Liveness Analysis

- A variable v is live at point p if
  - v is used along some path starting at p, and
  - no definition of v along the path before the use.
- When is a variable v dead at point p?
  - No use of v on any path from p to exit node, or
  - If all paths from p redefine v before using v.

What Use is Liveness Information?

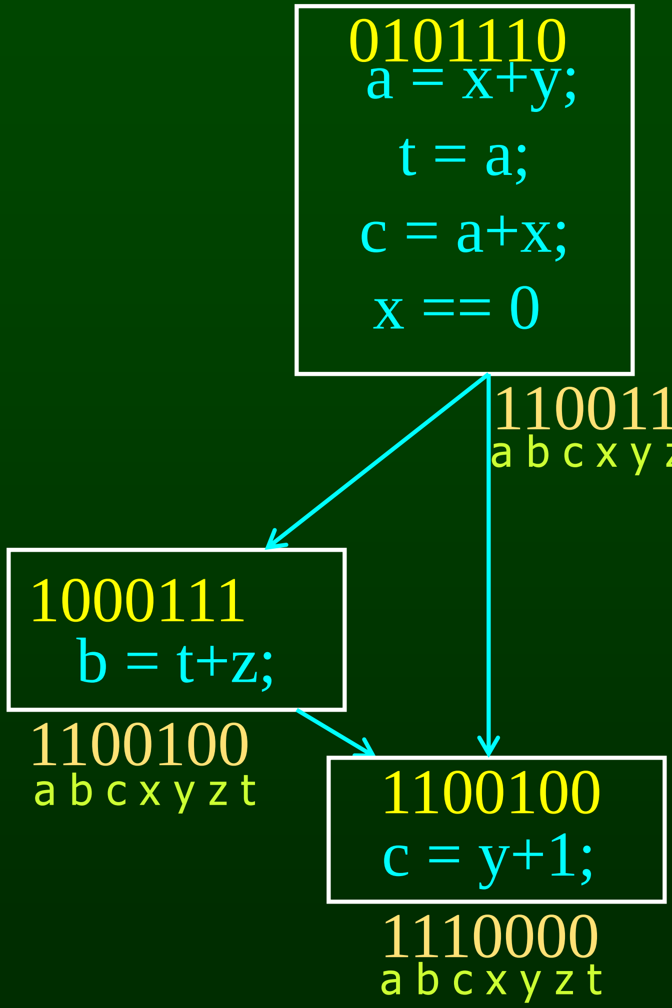
- Register allocation.
  - If a variable is dead, can reassign its register
- Dead code elimination.
  - Eliminate assignments to variables not read later.
  - But must not eliminate last assignment to variable (such as instance variable) visible outside CFG.
  - Can eliminate other dead assignments.
  - Handle by making all externally visible variables live on exit from CFG

Conceptual Idea of Analysis

- Simulate execution
- But start from exit and go backwards in CFG
- Compute liveness information from end to beginning of basic blocks

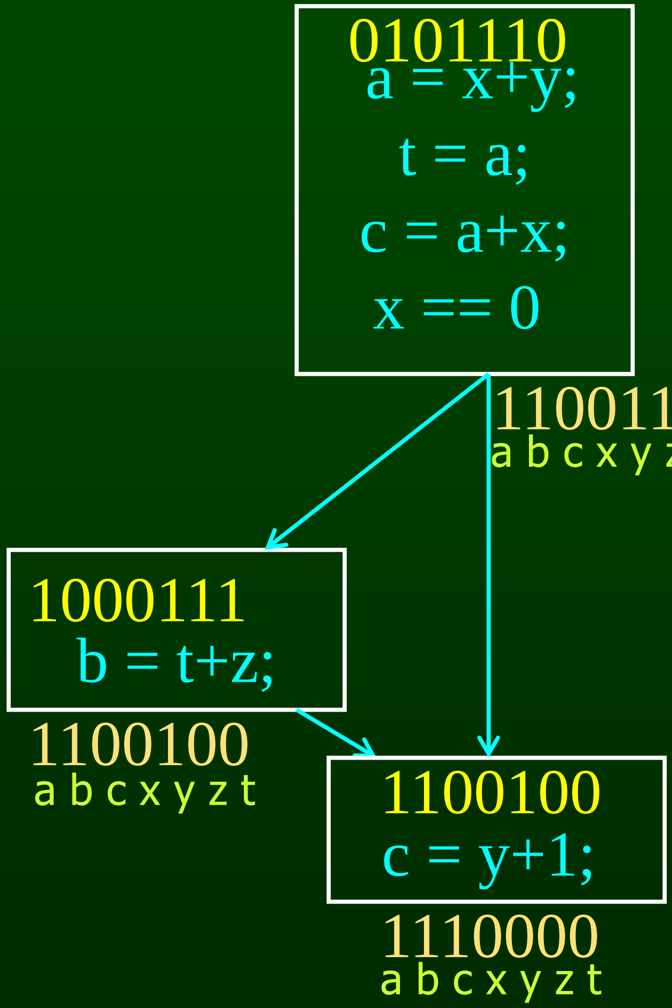
Liveness Example

- Assume a,b,c visible outside method
- So are live on exit
- Assume x,y,z,t not visible
- Represent Liveness Using Bit Vector
  - order is abcxyzt



Dead Code Elimination

- Assume a,b,c visible outside method
- So are live on exit
- Assume x,y,z,t not visible
- Represent Liveness Using Bit Vector
  - order is abcxyzt



0101110
a = x+y;
t = a;
c = a+x;
x == 0

1100111
a b c x y z t

1000111
b = t+z;

1100100
a b c x y z t

1100100
c = y+1;

1110000
a b c x y z t

Formalizing Analysis

- Each basic block has
  - IN - set of variables live at start of block
  - OUT - set of variables live at end of block
  - USE - set of variables with upwards exposed uses in block
  - DEF - set of variables defined in block
- USE[x = z; x = x+1;] = { z } (x not in USE)
- DEF[x = z; x = x+1;y = 1;] = {x, y}
- Compiler scans each basic block to derive USE and DEF sets

Algorithm

```
for all nodes n in N - { Exit }
    IN[n] = emptyset;
OUT[Exit] = emptyset;
IN[Exit] = use[Exit];
Changed = N - { Exit };

while (Changed != emptyset)
    choose a node n in Changed;
    Changed = Changed - { n };

    OUT[n] = emptyset;
    for all nodes s in successors(n)
        OUT[n] = OUT[n] U IN[p];

    IN[n] = use[n] U (out[n] - def[n]);

    if (IN[n] changed)
        for all nodes p in predecessors(n)
            Changed = Changed U { p };
```

Similar to Other Dataflow Algorithms

- Backwards analysis, not forwards
- Still have transfer functions
- Still have confluence operators
- Can generalize framework to work for both forwards and backwards analyses

Comparison

Reaching Definitions

```
for all nodes n in N
    OUT[n] = emptyset;
IN[Entry] = emptyset;
OUT[Entry] = GEN[Entry];
Changed = N - { Entry };

while (Changed != emptyset)
    choose a node n in Changed;
    Changed = Changed - { n };

    IN[n] = emptyset;
    for all nodes p in predecessors(n)
        IN[n] = IN[n] U OUT[p];

    OUT[n] = GEN[n] U (IN[n] - KILL[n]);

    if (OUT[n] changed)
        for all nodes s in successors(n)
            Changed = Changed U { s };
```

Available Expressions

```
for all nodes n in N
    OUT[n] = E;
IN[Entry] = emptyset;
OUT[Entry] = GEN[Entry];
Changed = N - { Entry };

while (Changed != emptyset)
    choose a node n in Changed;
    Changed = Changed - { n };

    IN[n] = E;
    for all nodes p in predecessors(n)
        IN[n] = IN[n] ∩ OUT[p];

    OUT[n] = GEN[n] U (IN[n] - KILL[n]);

    if (OUT[n] changed)
            for all nodes s in successors(n)
            Changed = Changed U { s };
```

Liveness

```
for all nodes n in N - { Exit }
    IN[n] = emptyset;
OUT[Exit] = emptyset;
IN[Exit] = use[Exit];
Changed = N - { Exit };

while (Changed != emptyset)
    choose a node n in Changed;
    Changed = Changed - { n };

    OUT[n] = emptyset;
    for all nodes s in successors(n)
        OUT[n] = OUT[n] U IN[p];

    IN[n] = use[n] U (out[n] - def[n]);

    if (IN[n] changed)
        for all nodes p in predecessors(n)
            Changed = Changed U { p };
```

Comparison

Reaching Definitions

```
for all nodes n in N
    OUT[n] = emptyset;
IN[Entry] = emptyset;
OUT[Entry] = GEN[Entry];
Changed = N - { Entry };

while (Changed != emptyset)
    choose a node n in Changed;
    Changed = Changed - { n };

    IN[n] = emptyset;
    for all nodes p in predecessors(n)
        IN[n] = IN[n] U OUT[p];

    OUT[n] = GEN[n] U (IN[n] - KILL[n]);

    if (OUT[n] changed)
        for all nodes s in successors(n)
            Changed = Changed U { s };
```

Available Expressions

```
for all nodes n in N
    OUT[n] = E;
IN[Entry] = emptyset;
OUT[Entry] = GEN[Entry];
Changed = N - { Entry };

while (Changed != emptyset)
    choose a node n in Changed;
    Changed = Changed - { n };

    IN[n] = E;
    for all nodes p in predecessors(n)
        IN[n] = IN[n] ∩ OUT[p];

    OUT[n] = GEN[n] U (IN[n] - KILL[n]);

    if (OUT[n] changed)
            for all nodes s in successors(n)
            Changed = Changed U { s };
```

Comparison

Reaching Definitions

```
for all nodes n in N
    OUT[n] = emptyset;
IN[Entry] = emptyset;
OUT[Entry] = GEN[Entry];
Changed = N - { Entry };

while (Changed != emptyset)
    choose a node n in Changed;
    Changed = Changed - { n };

    IN[n] = emptyset;
    for all nodes p in predecessors(n)
        IN[n] = IN[n] U OUT[p];

    OUT[n] = GEN[n] U (IN[n] - KILL[n]);

    if (OUT[n] changed)
        for all nodes s in successors(n)
            Changed = Changed U { s };
```

Liveness

```
for all nodes n in N
    IN[n] = emptyset;
OUT[Exit] = emptyset;
IN[Exit] = use[Exit];
Changed = N - { Exit };

while (Changed != emptyset)
    choose a node n in Changed;
    Changed = Changed - { n };

    OUT[n] = emptyset;
    for all nodes s in successors(n)
        OUT[n] = OUT[n] U IN[p];

    IN[n] = use[n] U (out[n] - def[n]);

    if (IN[n] changed)
        for all nodes p in predecessors(n)
            Changed = Changed U { p };
```

Analysis Information Inside Basic Blocks

- One detail:
  - Given dataflow information at IN and OUT of node
  - Also need to compute information at each statement of basic block
  - Simple propagation algorithm usually works fine
  - Can be viewed as restricted case of dataflow analysis

Pessimistic vs. Optimistic Analyses

- Available expressions is optimistic (for common sub-expression elimination)
  - Assume expressions are available at start of analysis
  - Analysis eliminates all that are not available
  - Cannot stop analysis early and use current result
- Live variables is pessimistic (for dead code elimination)
  - Assume all variables are live at start of analysis
  - Analysis finds variables that are dead
  - Can stop analysis early and use current result
- Dataflow setup same for both analyses
- Optimism/pessimism depends on intended use

Summary

- Basic Blocks and Basic Block Optimizations
  - Copy and constant propagation
  - Common sub-expression elimination
  - Dead code elimination

- Dataflow Analysis
  - Control flow graph
  - IN[b], OUT[b], transfer functions, join points

- Paired analyses and transformations
  - Reaching definitions/constant propagation
  - Available expressions/common sub-expression elimination
  - Liveness analysis/Dead code elimination

- Stacked analysis and transformations work together