6.035

Introduction to Dataflow Analysis

Value Numbering Summary

- Forward symbolic execution of basic block
- Maps
  - Var2Val – symbolic value for each variable
  - Exp2Val – value of each evaluated expression
  - Exp2Tmp – tmp that holds value of each evaluated expression
- Algorithm
  - For each statement
    - If variables in RHS not in the Var2Val add it with a new value
    - If RHS expression in Exp2Tmp use that Temp
    - If not add RHS expression to Exp2Val with new value
    - Copy the value into a new tmp and add to EXp2Tmp

Copy Propagation Summary

- Forward Propagation within basic block
- Maps
  - tmp2var: tells which variable to use instead of a given temporary variable
  - var2set: inverse of tmp to var. tells which temps are mapped to a given variable by tmp to var
- Algorithm
  - For each statement
    - If any tmp variable in the RHS is in tmp2var replace it with var
    - If LHS var in var2set remove the variables in the set in tmp2var

Dead Code Elimination Summary

- Backward Propagation within basic block
- Map
  - A set of variables that are needed later in computation
- Algorithm
  - Every statement encountered
    - If LHS is not in the set, remove the statement
    - Else put all the variables in the RHS into the set

Summary So far… what's next

- Till now: How to analyze and transform within a basic block

- Next: How to do it for the entire procedure

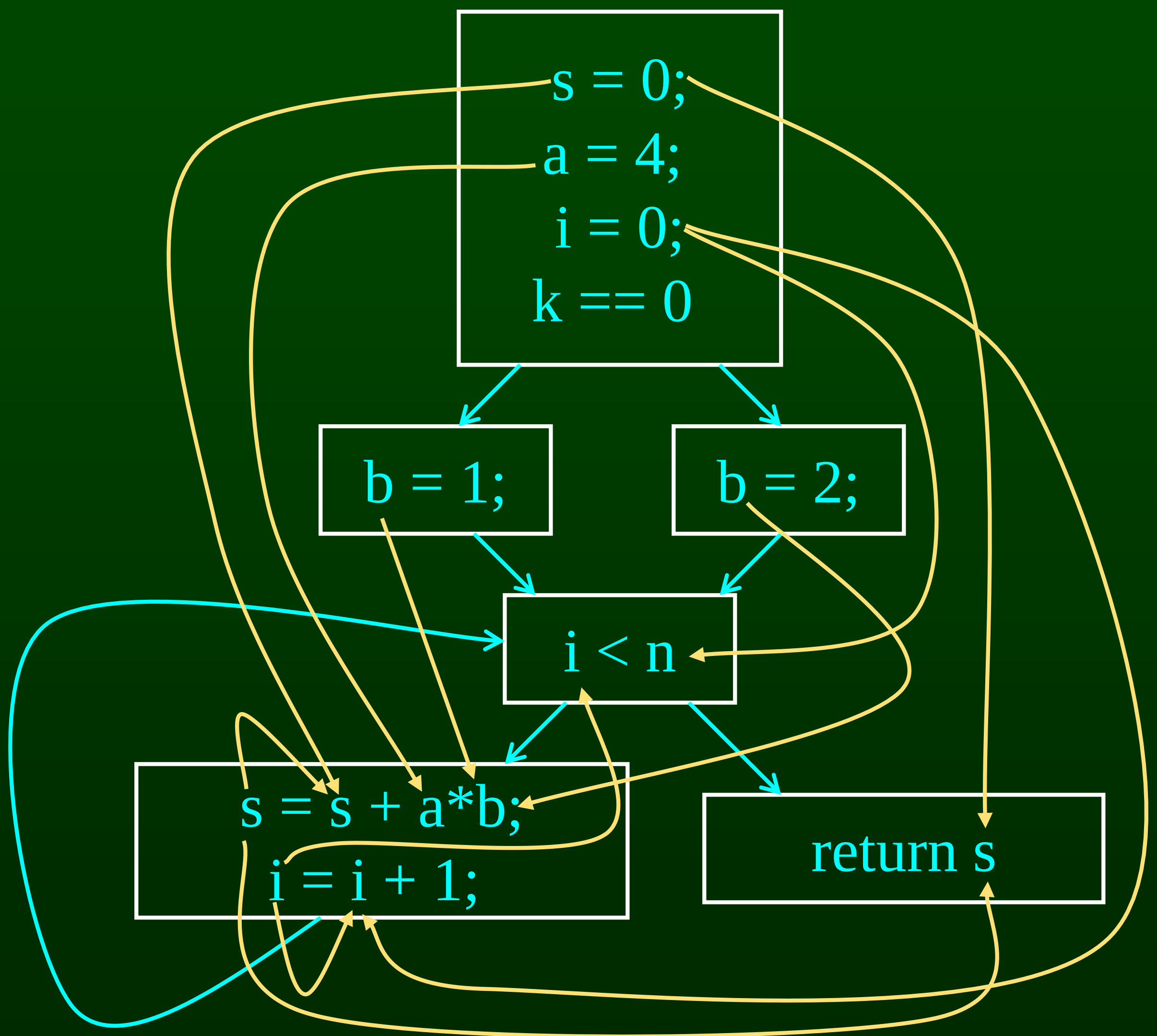
Outline

- Reaching Definitions
- Available Expressions
- Liveness

Reaching Definitions

- Concept of definition and use
  - a = x+y
  - is a definition of a
  - is a use of x and y
- A definition reaches a use if
  - value written by definition
  - may be read by use

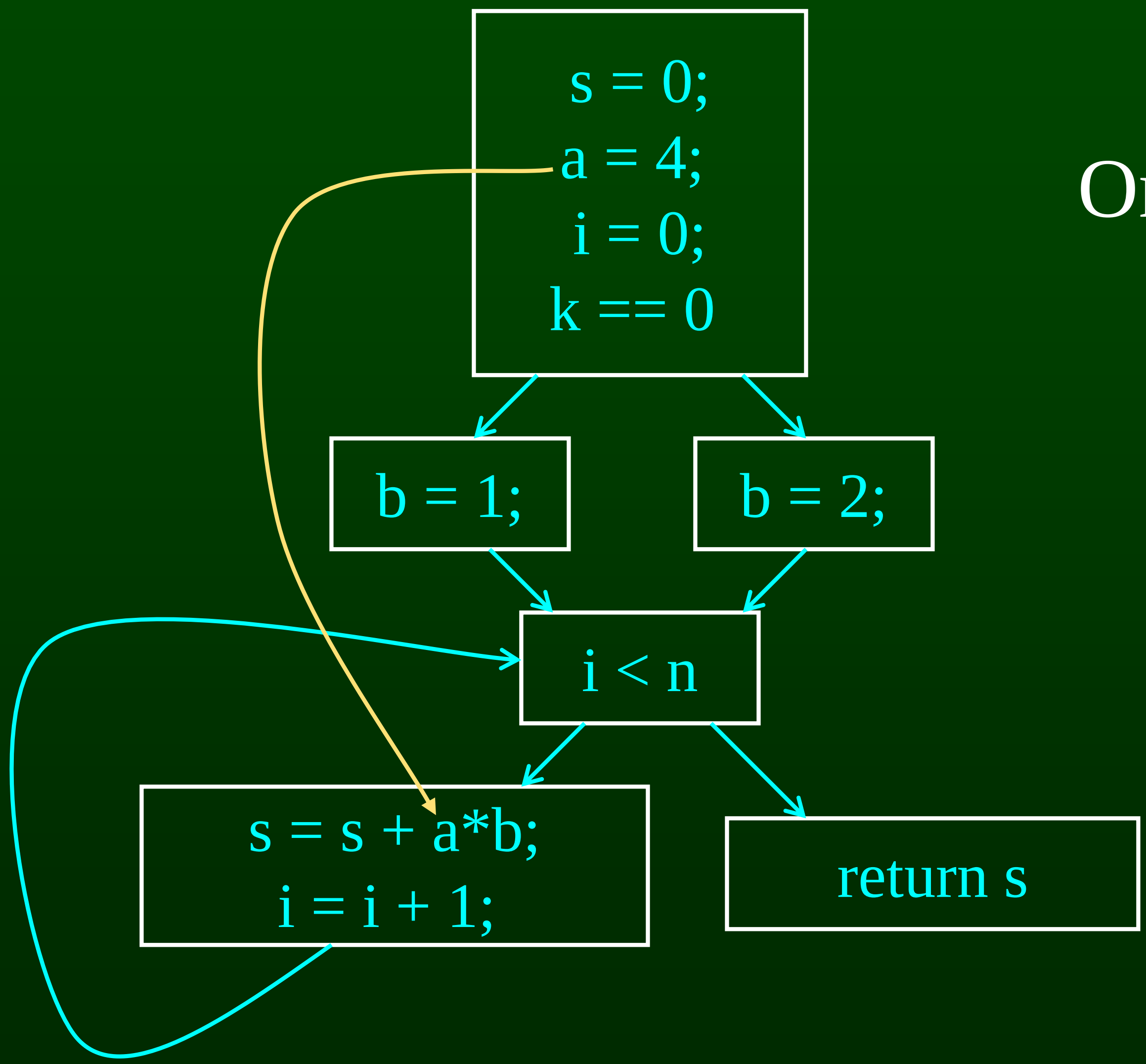
Reaching Definitions



Reaching Definitions and Constant Propagation

- Is a use of a variable a constant?
  - Check all reaching definitions
  - If all assign variable to same constant
  - Then use is in fact a constant
- Can replace variable with constant

Is a Constant in s = s+a*b?

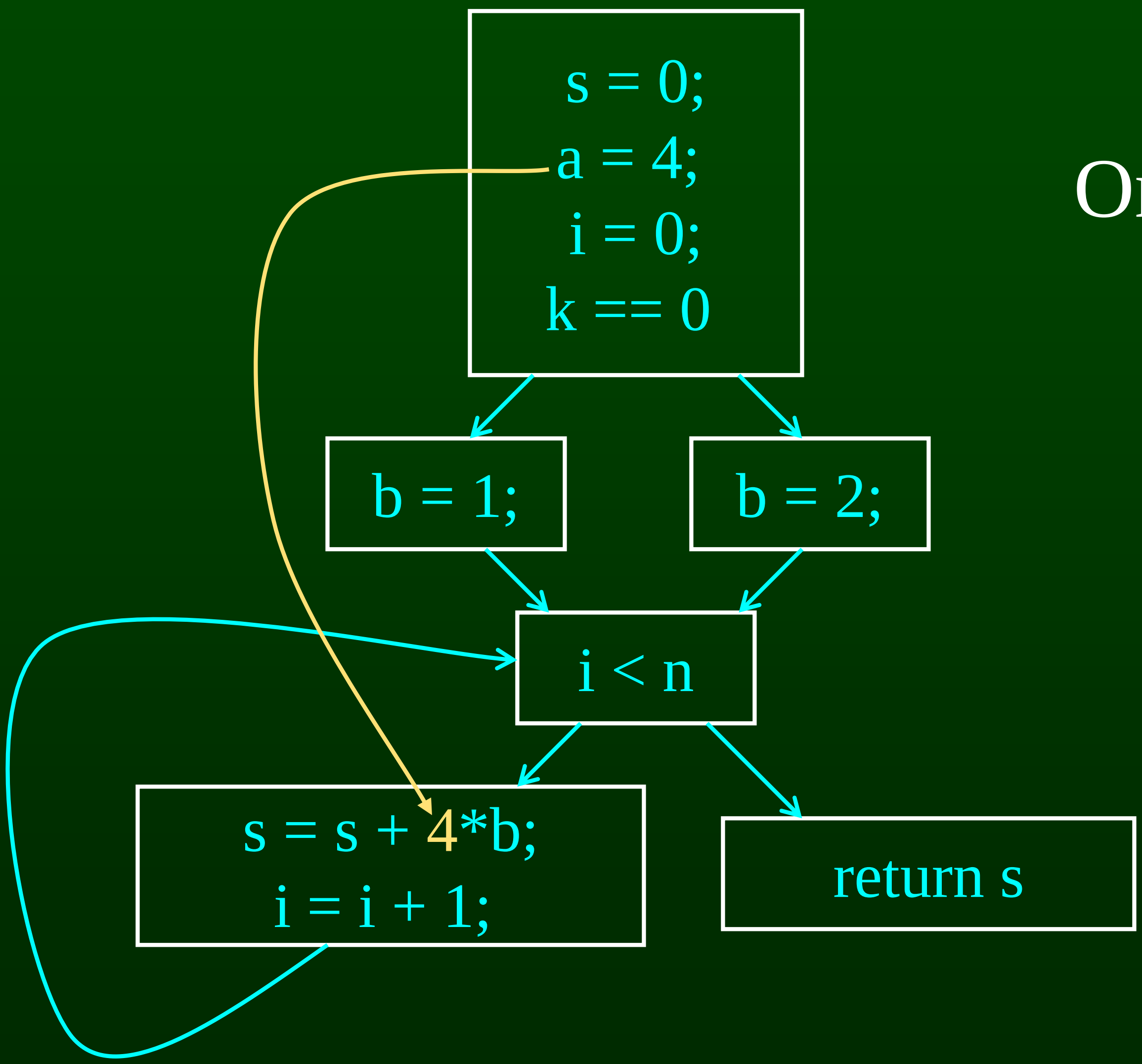


Yes!

On all reaching definitions

a = 4

Constant Propagation Transform

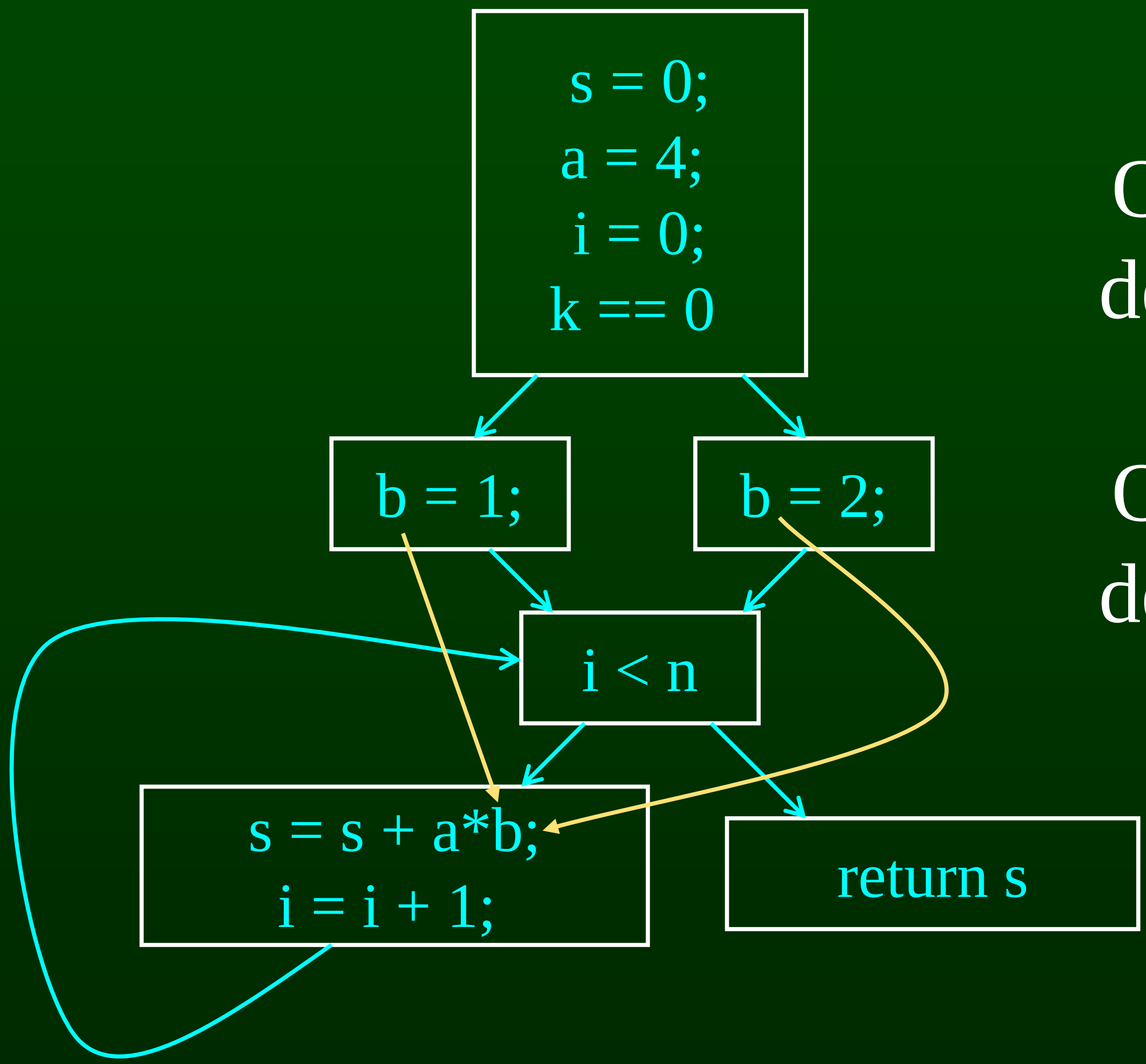


Yes!

On all reaching definitions

a = 4

Is b Constant in s = s+a*b?

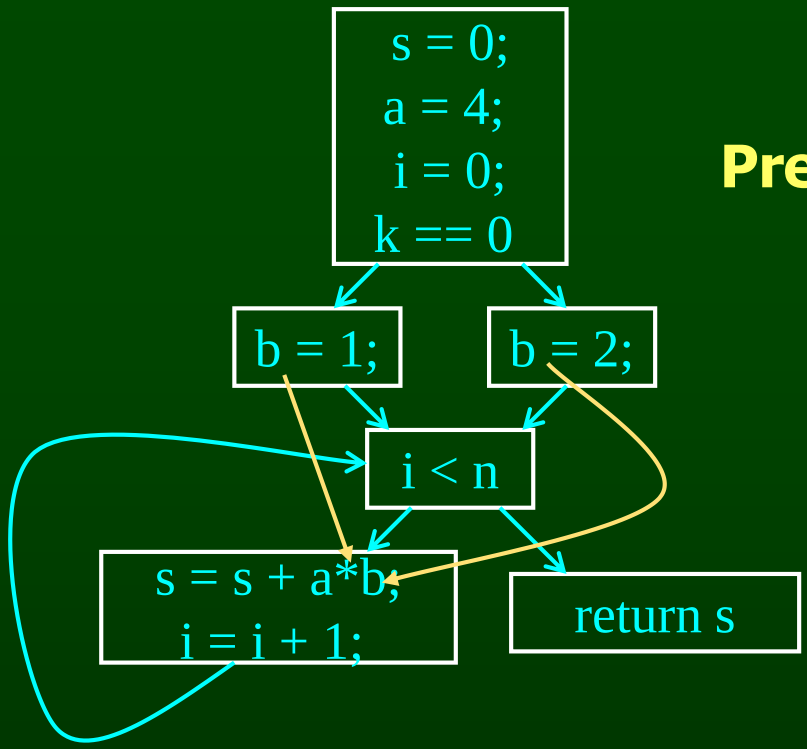


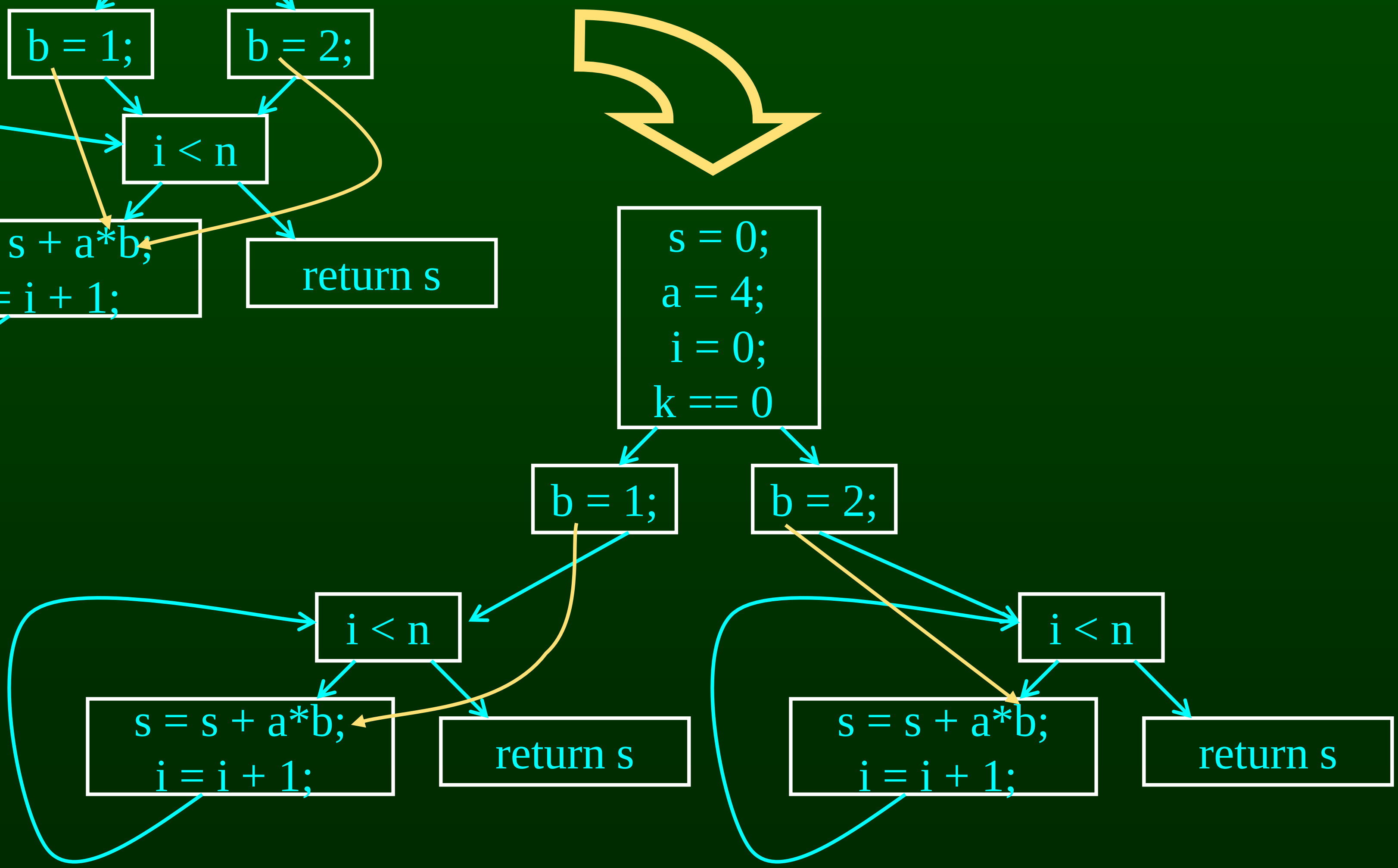
No!

One reaching definition with b = 1

One reaching definition with b = 2

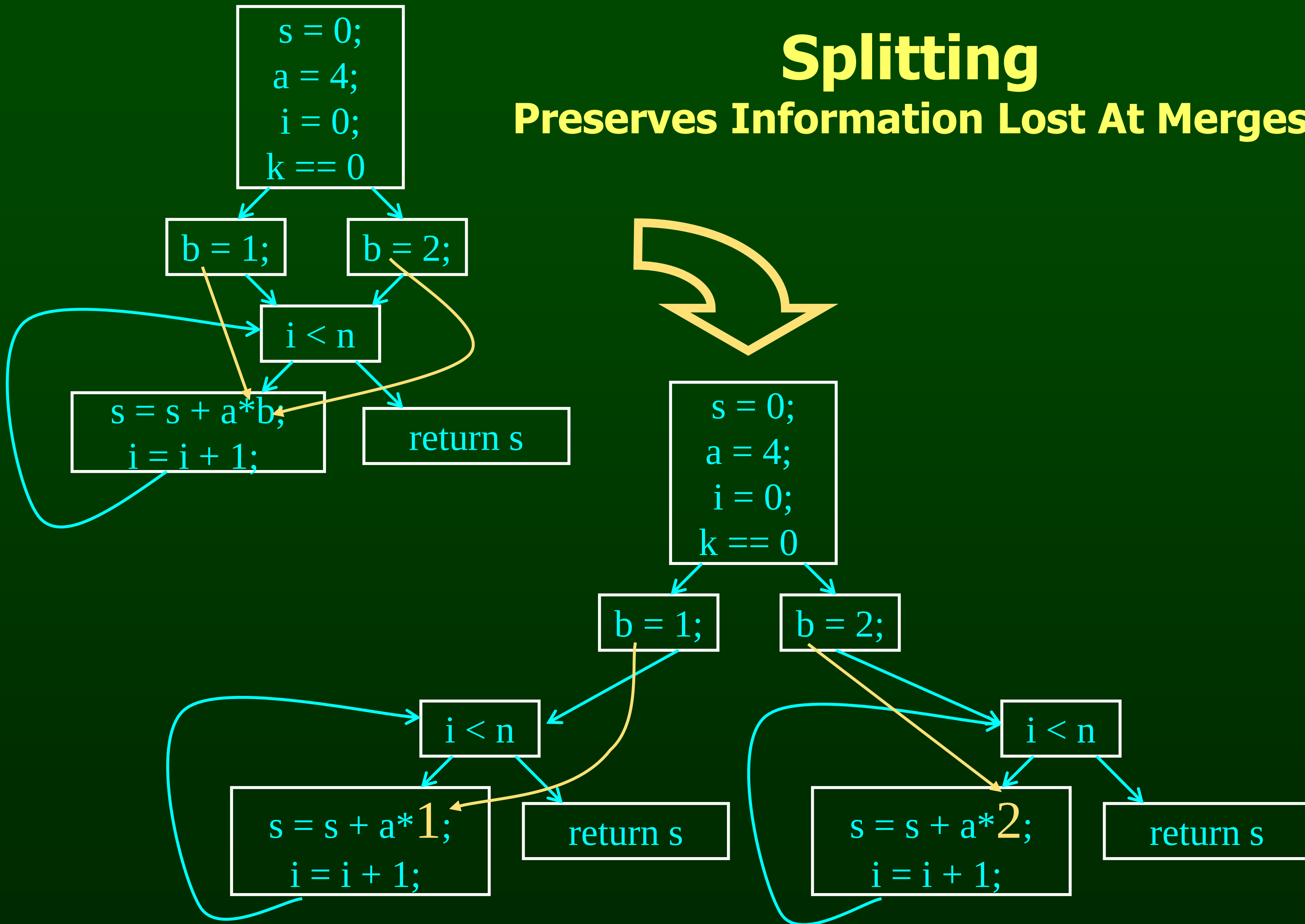
Splitting
Preserves Information Lost At Merges





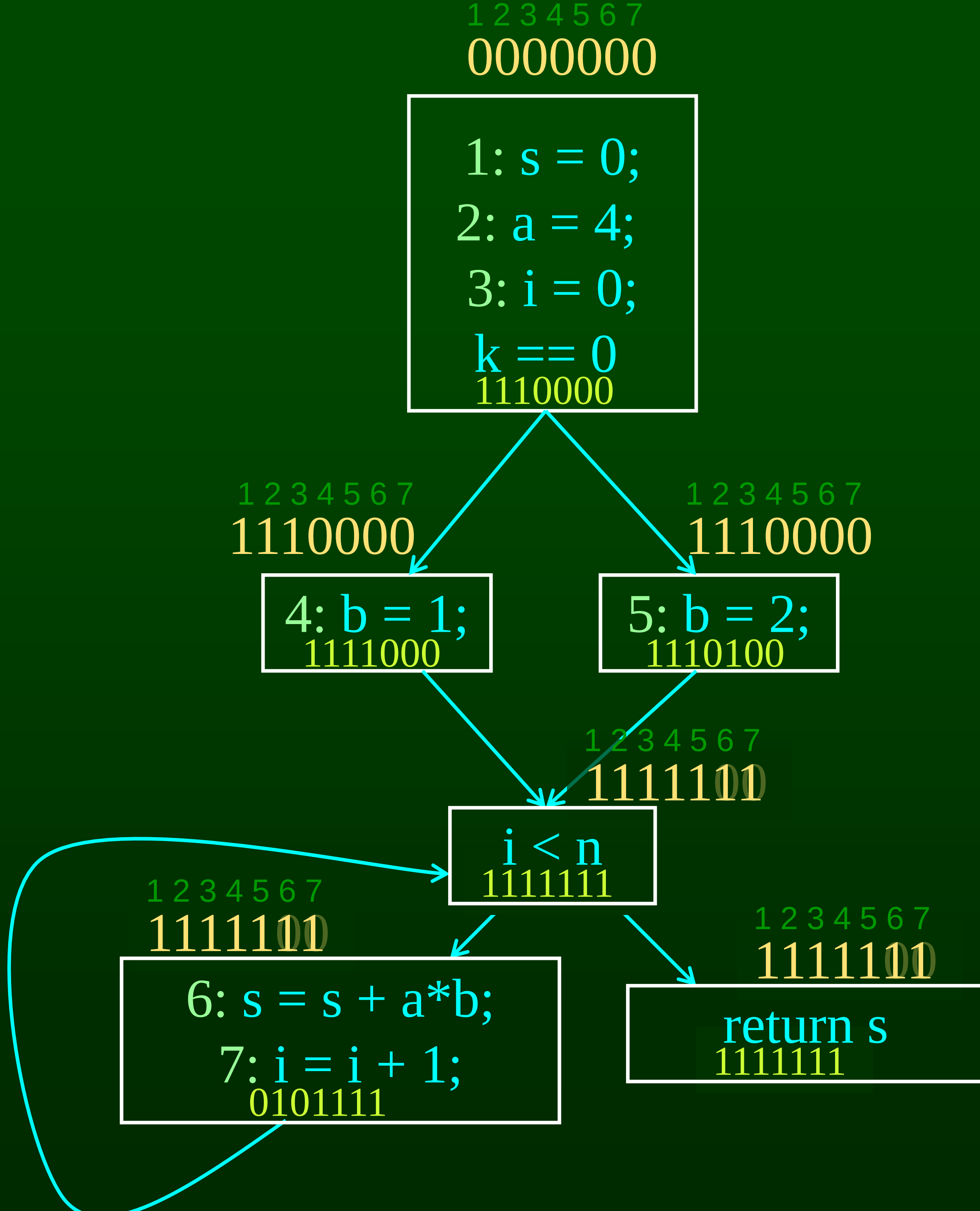
Splitting

Preserves Information Lost At Merges



Computing Reaching Definitions

- Compute with sets of definitions
  - represent sets using bit vectors
  - each definition has a position in bit vector
- At each basic block, compute
  - definitions that reach start of block
  - definitions that reach end of block
- Do computation by simulating execution of program until reach fixed point



1 2 3 4 5 6 7
0000000
1: s = 0;
2: a = 4;
3: i = 0;
k == 0
1110000
1 2 3 4 5 6 7
1110000
4: b = 1;
1111000
1 2 3 4 5 6 7
1110000
5: b = 2;
1110100
1 2 3 4 5 6 7
1111111
i < n
1111111
1 2 3 4 5 6 7
1111111
6: s = s + a*b;
7: i = i + 1;
0101111
1 2 3 4 5 6 7
1111111
return s
1111111

Formalizing Analysis

- Each basic block has
  - IN - set of definitions that reach beginning of block
  - OUT - set of definitions that reach end of block
  - GEN - set of definitions generated in block
  - KILL - set of definitions killed in block
- GEN[s = s + a*b; i = i + 1;] = 0000011
- KILL[s = s + a*b; i = i + 1;] = 1010000
- Compiler scans each basic block to derive GEN and KILL sets

Dataflow Equations

- IN[b] = OUT[b1] U ... U OUT[bn]
  - where b1, ..., bn are predecessors of b in CFG
- OUT[b] = (IN[b] - KILL[b]) U GEN[b]
- IN[entry] = 0000000
- Result: system of equations

Solving Equations

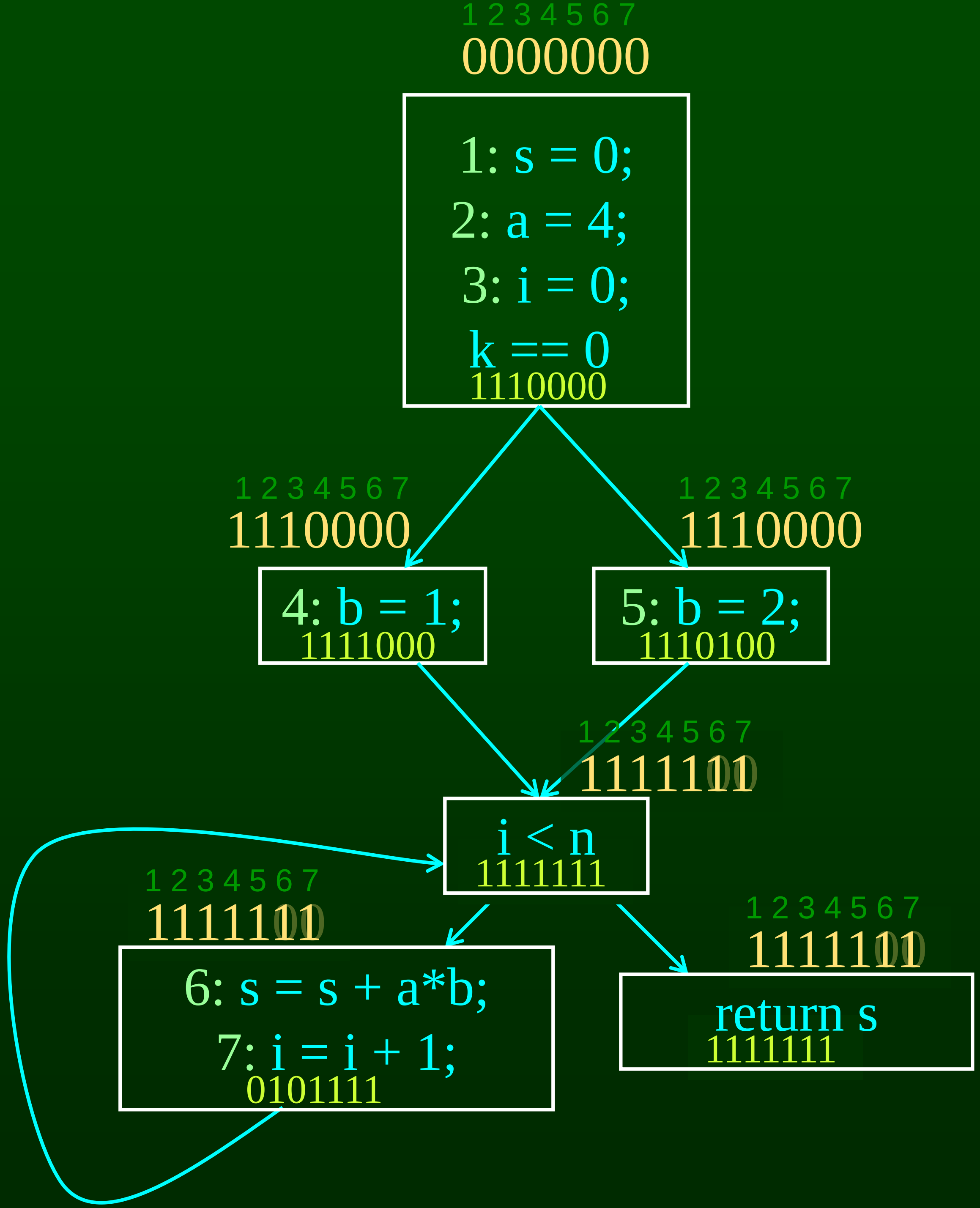
- Use fixed point algorithm
- Initialize with solution of OUT[b] = 0000000
- Repeatedly apply equations
  - IN[b] = OUT[b1] U ... U OUT[bn]
  - OUT[b] = (IN[b] - KILL[b]) U GEN[b]
- Until reach fixed point
- Until equation application has no further effect
- Use a worklist to track which equation applications may have a further effect

Reaching Definitions Algorithm

```
for all nodes n in N
      OUT[n] = emptyset; // OUT[n] = GEN[n];
IN[Entry] = emptyset;
OUT[Entry] = GEN[Entry];
Changed = N - { Entry }; // N = all nodes in graph

while (Changed != emptyset)
      choose a node n in Changed;
      Changed = Changed - { n };

      IN[n] = emptyset;
      for all nodes p in predecessors(n)
            IN[n] = IN[n] U OUT[p];

      OUT[n] = GEN[n] U (IN[n] - KILL[n]);

      if (OUT[n] changed)
            for all nodes s in successors(n)
                  Changed = Changed U { s };
```

Questions

- Does the algorithm halt?
  - yes, because transfer function is monotonic
  - if increase IN, increase OUT
  - in limit, all bits are 1
- If bit is 0, does the corresponding definition ever reach basic block?
- If bit is 1, is does the corresponding definition always reach the basic block?



1 2 3 4 5 6 7
0000000
1: s = 0;
2: a = 4;
3: i = 0;
k == 0
1110000
1 2 3 4 5 6 7
1110000
4: b = 1;
1111000
1 2 3 4 5 6 7
1110000
5: b = 2;
1110100
1 2 3 4 5 6 7
1111111
i < n
1111111
1 2 3 4 5 6 7
1111111
6: s = s + a*b;
7: i = i + 1;
0101111
1 2 3 4 5 6 7
1111111
return s
1111111

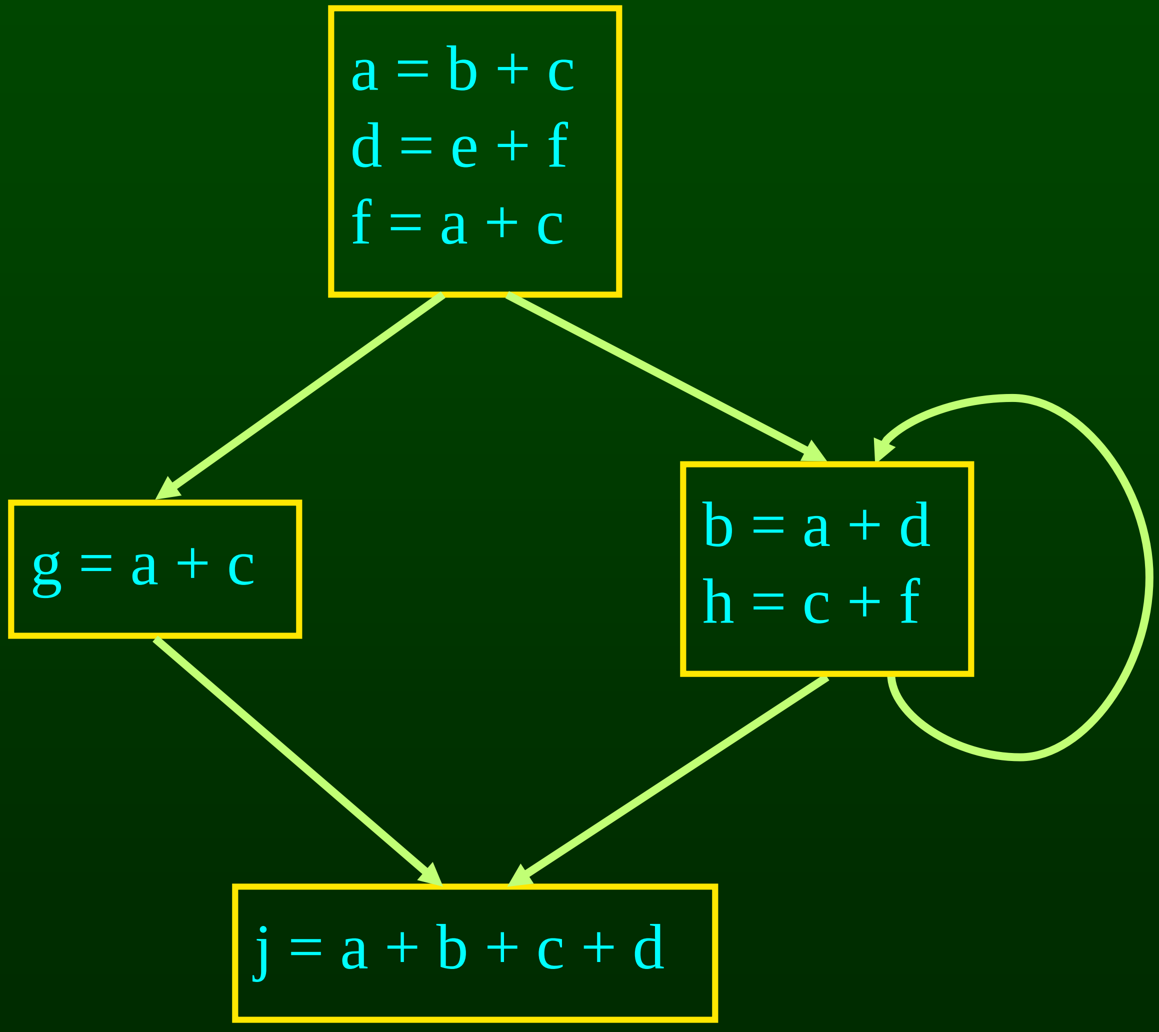
Outline

- Reaching Definitions
- **Available Expressions**
- Liveness

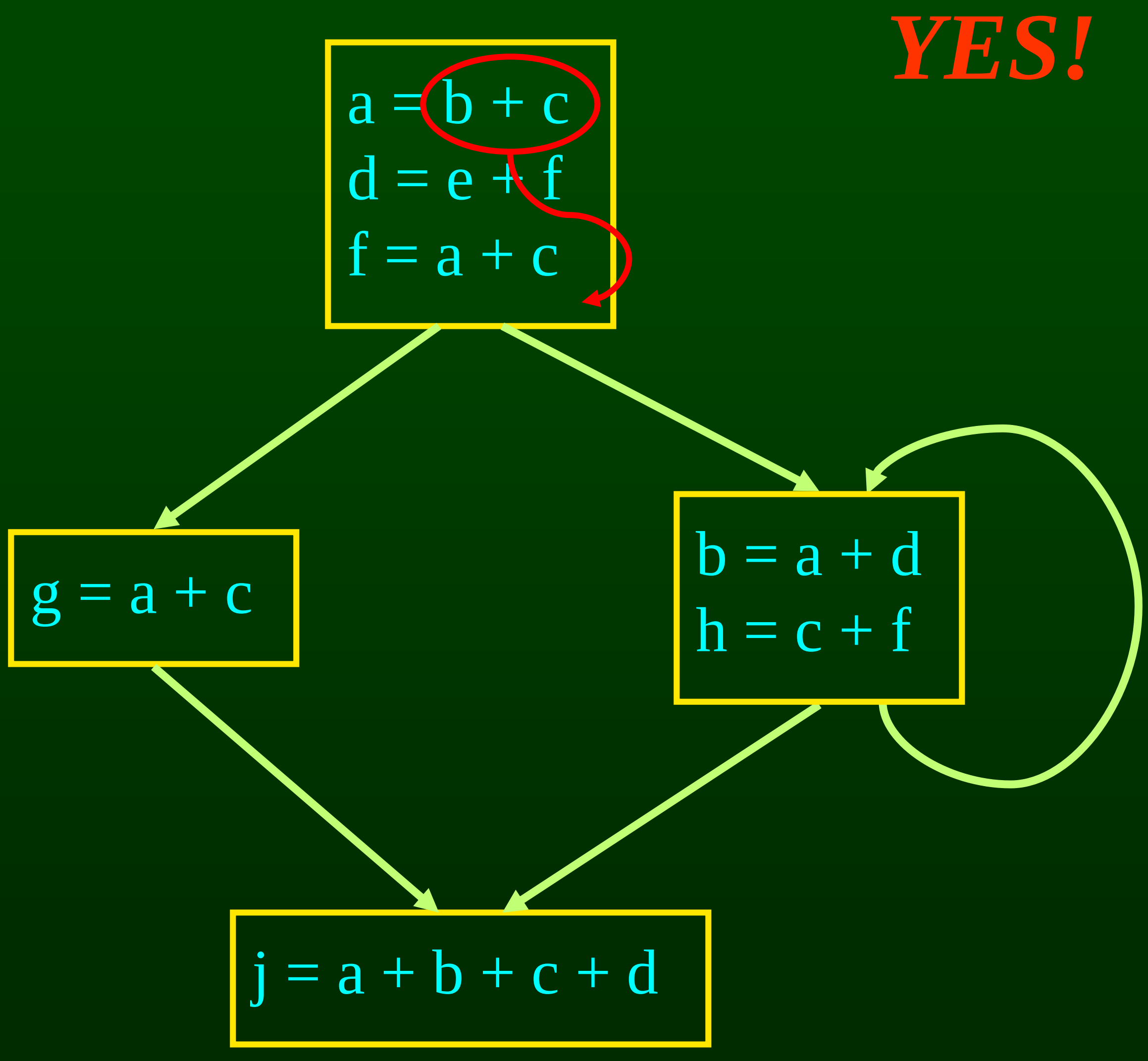
Available Expressions

- An expression x+y is available at a point p if
  - every path from the initial node to p must evaluate x+y before reaching p,
  - and there are no assignments to x or y after the evaluation but before p.
- Available Expression information can be used to do global (across basic blocks) CSE
- If expression is available at use, no need to reevaluate it

Example: Available Expression

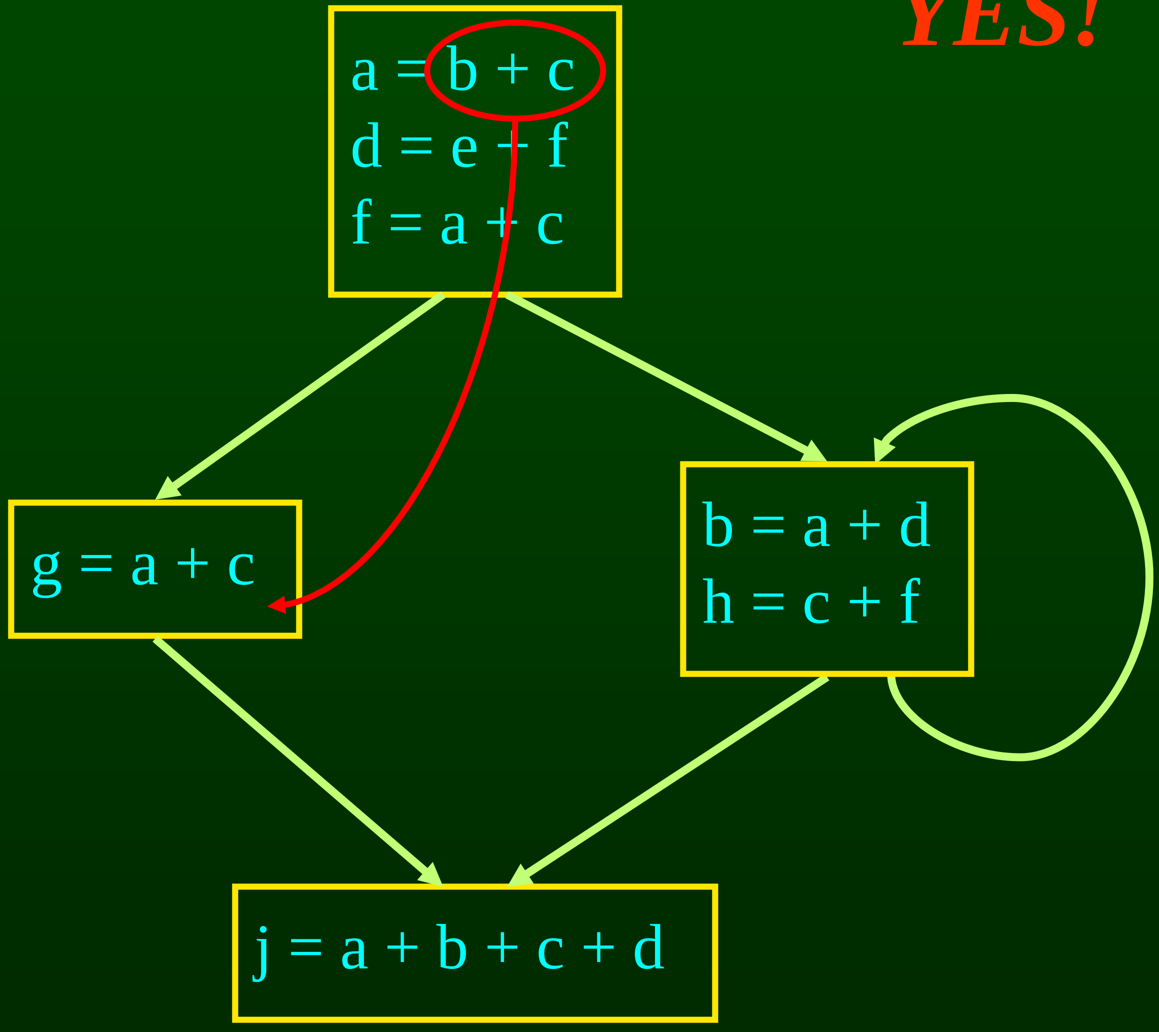


Is the Expression Available?



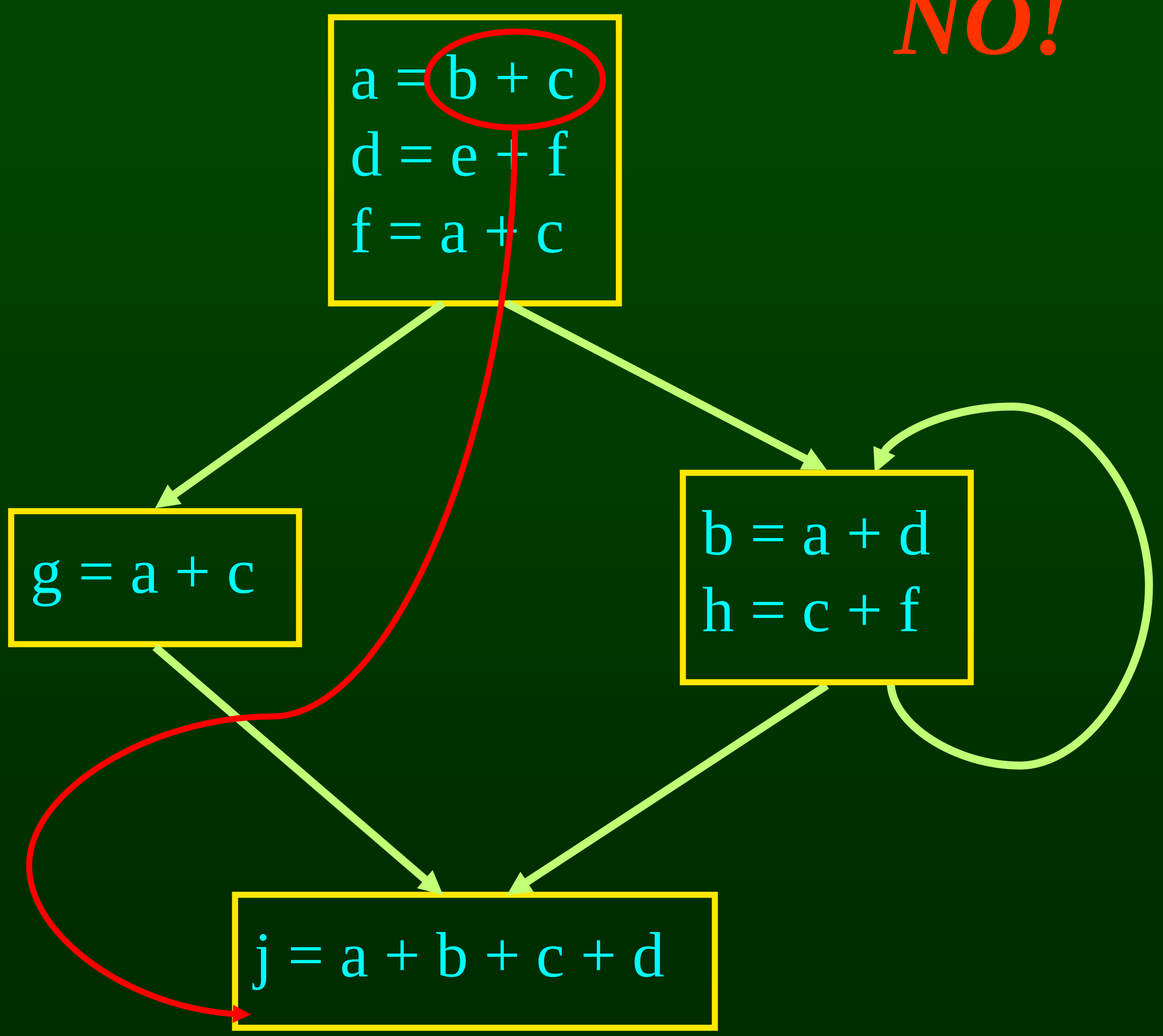
Is the Expression Available?

YES!

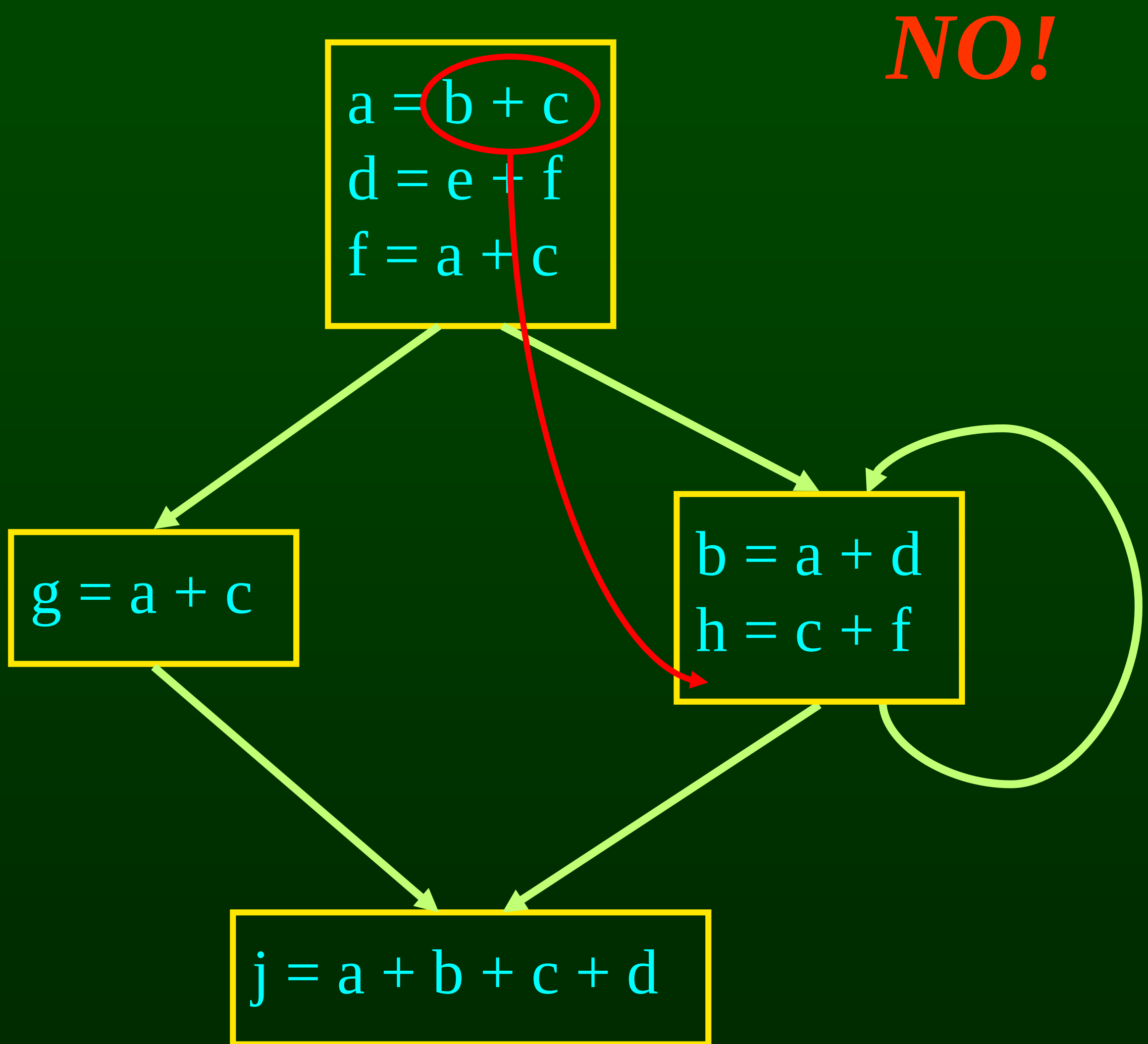


Is the Expression Available?

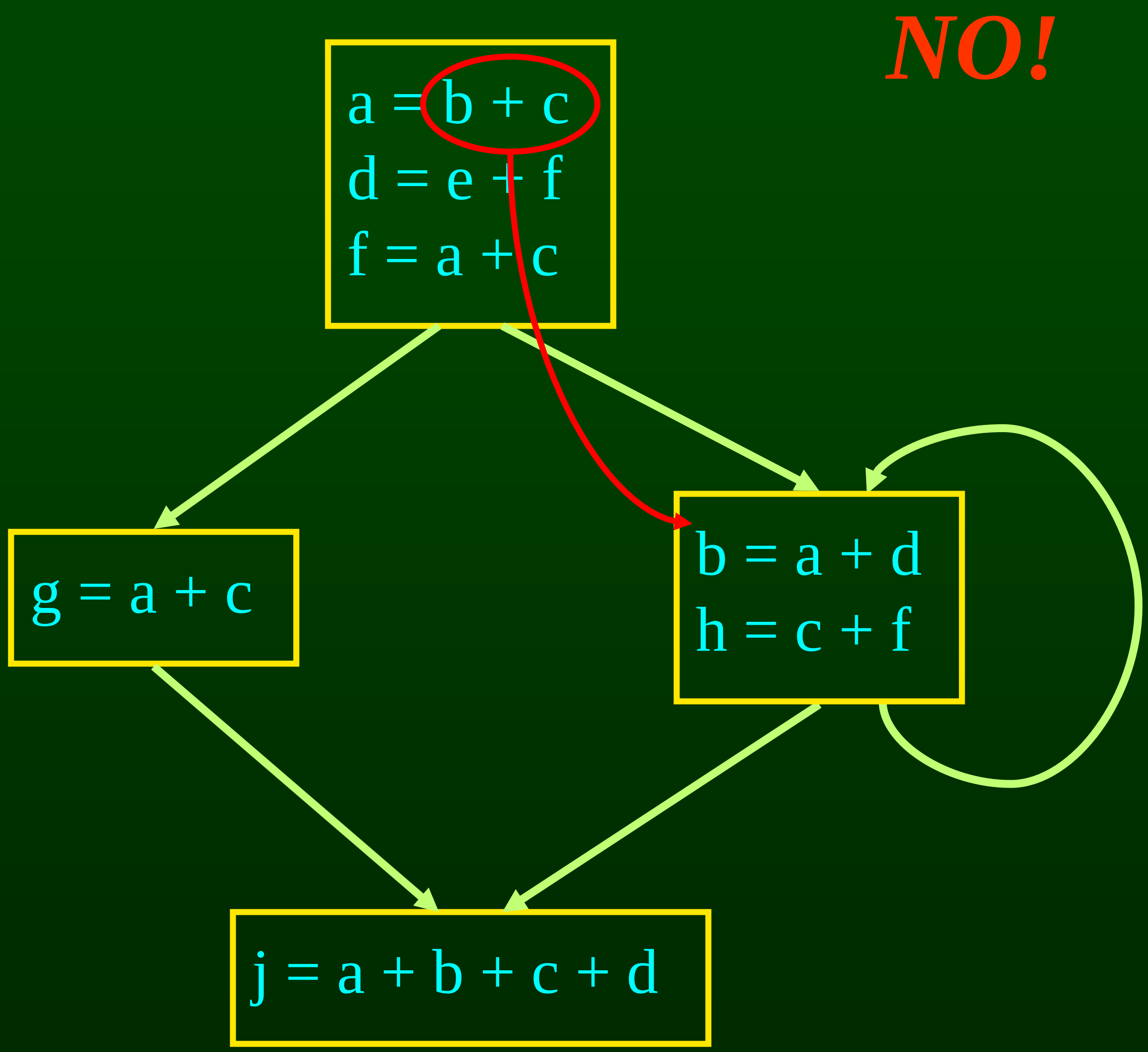
NO!



Is the Expression Available?

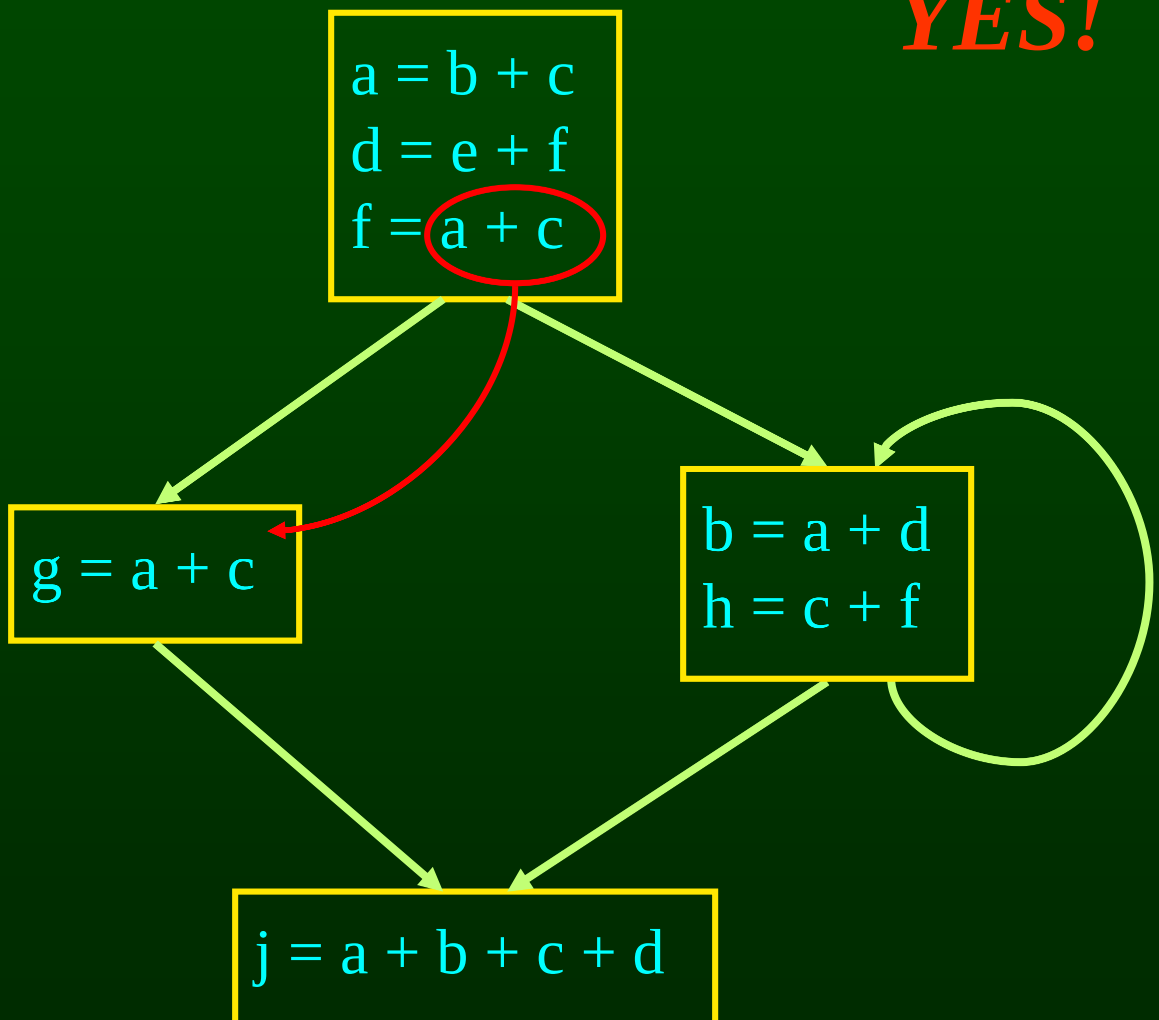


Is the Expression Available?



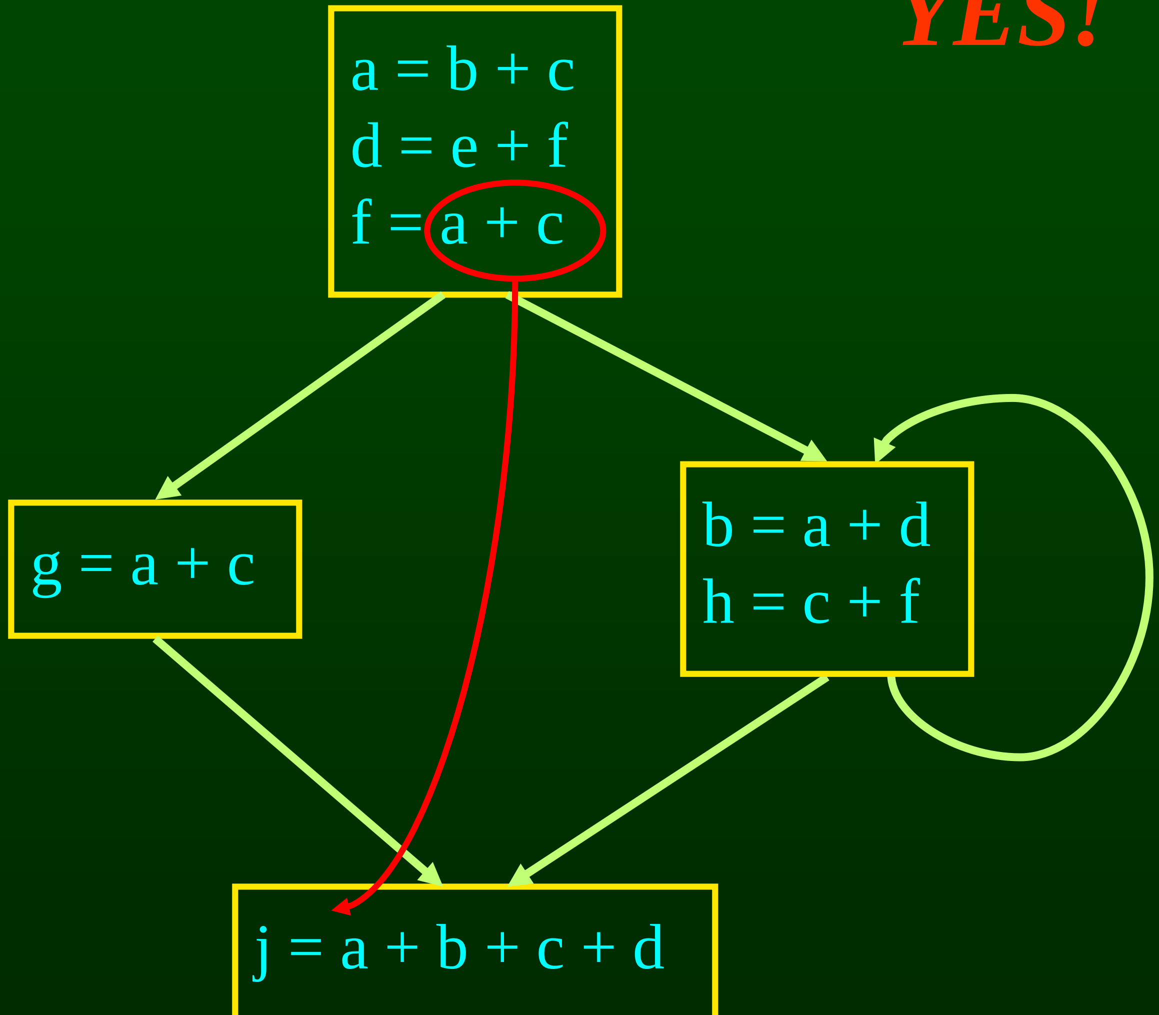
Is the Expression Available?

YES!

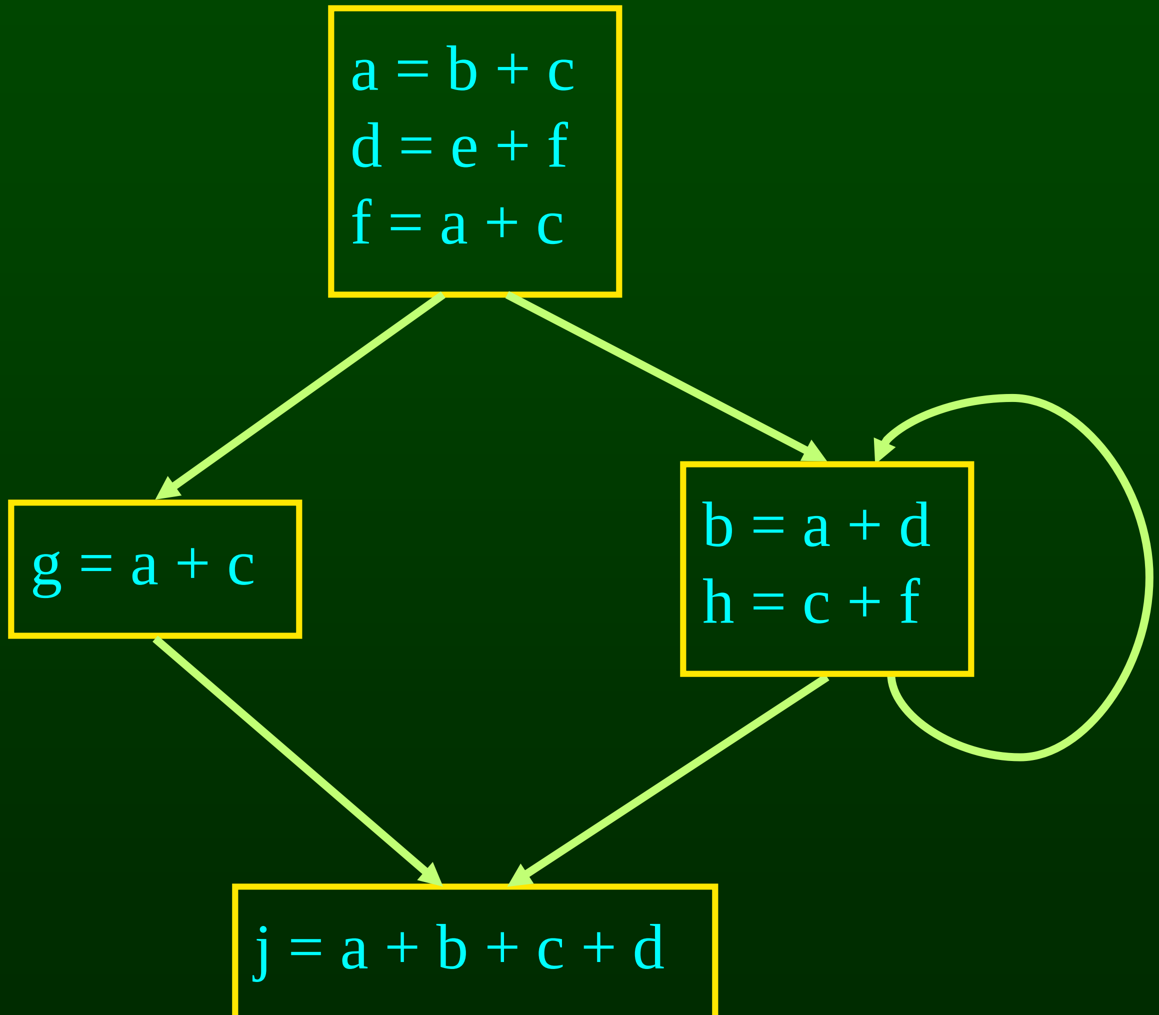


Is the Expression Available?

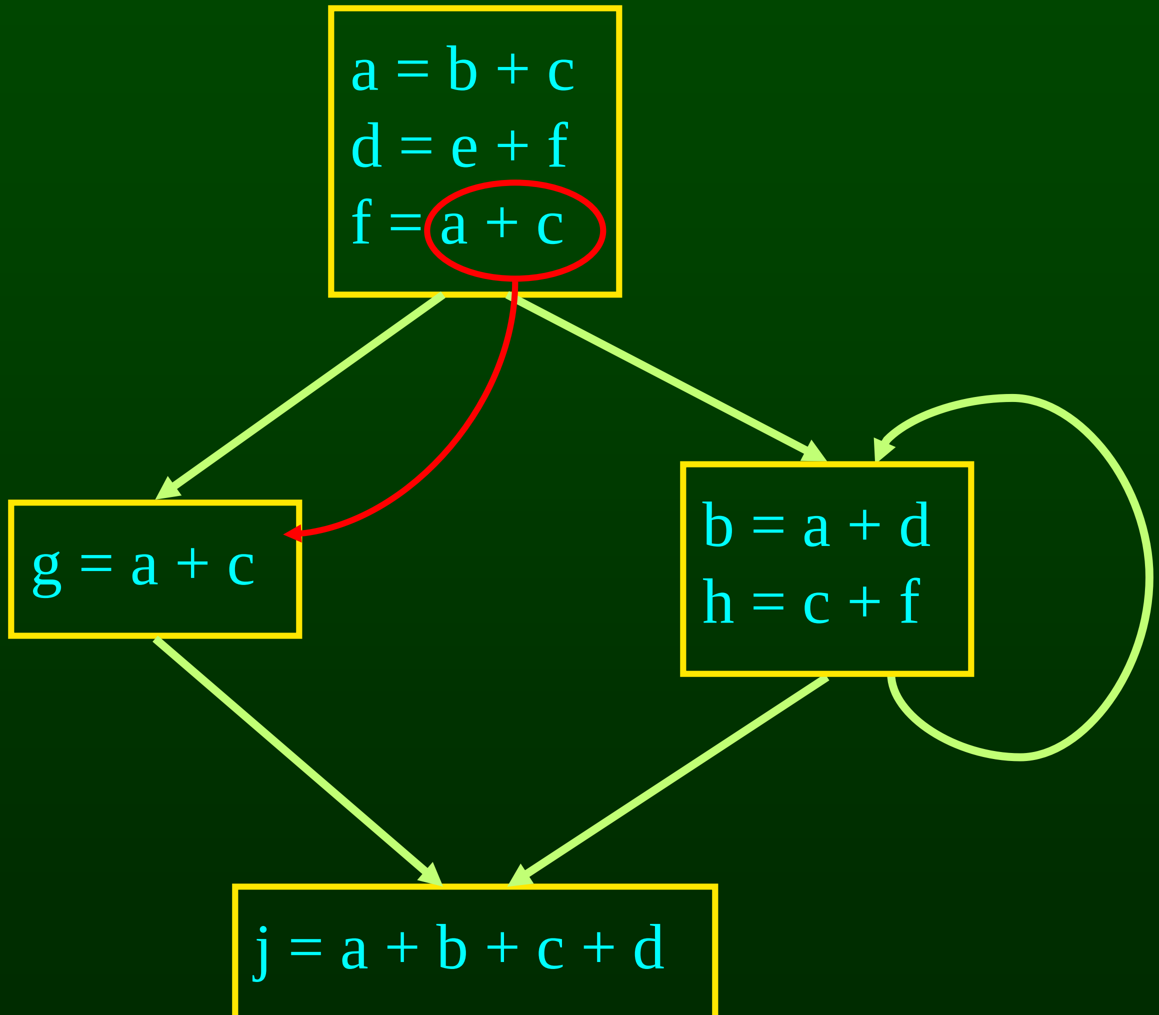
YES!



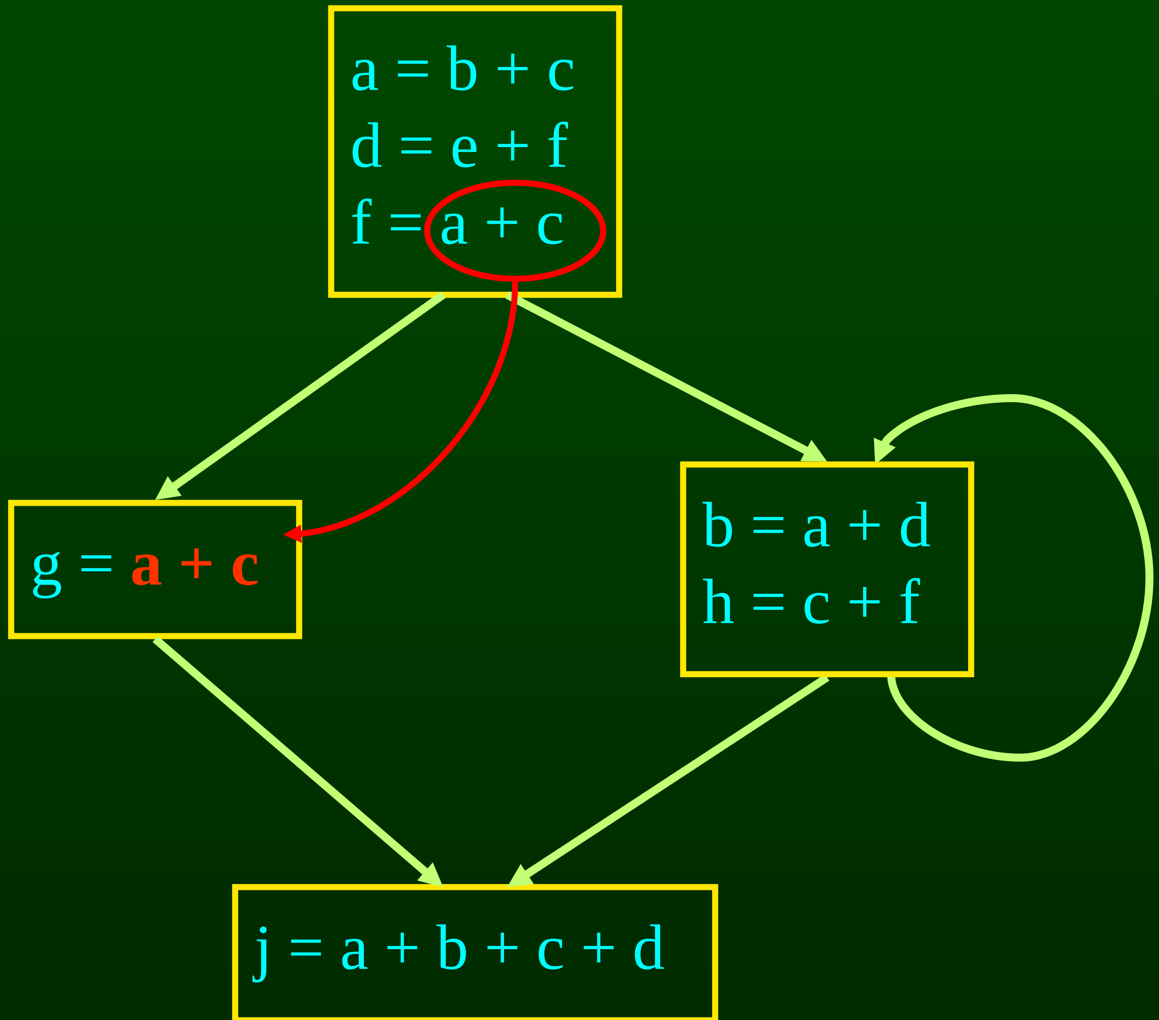
Use of Available Expressions



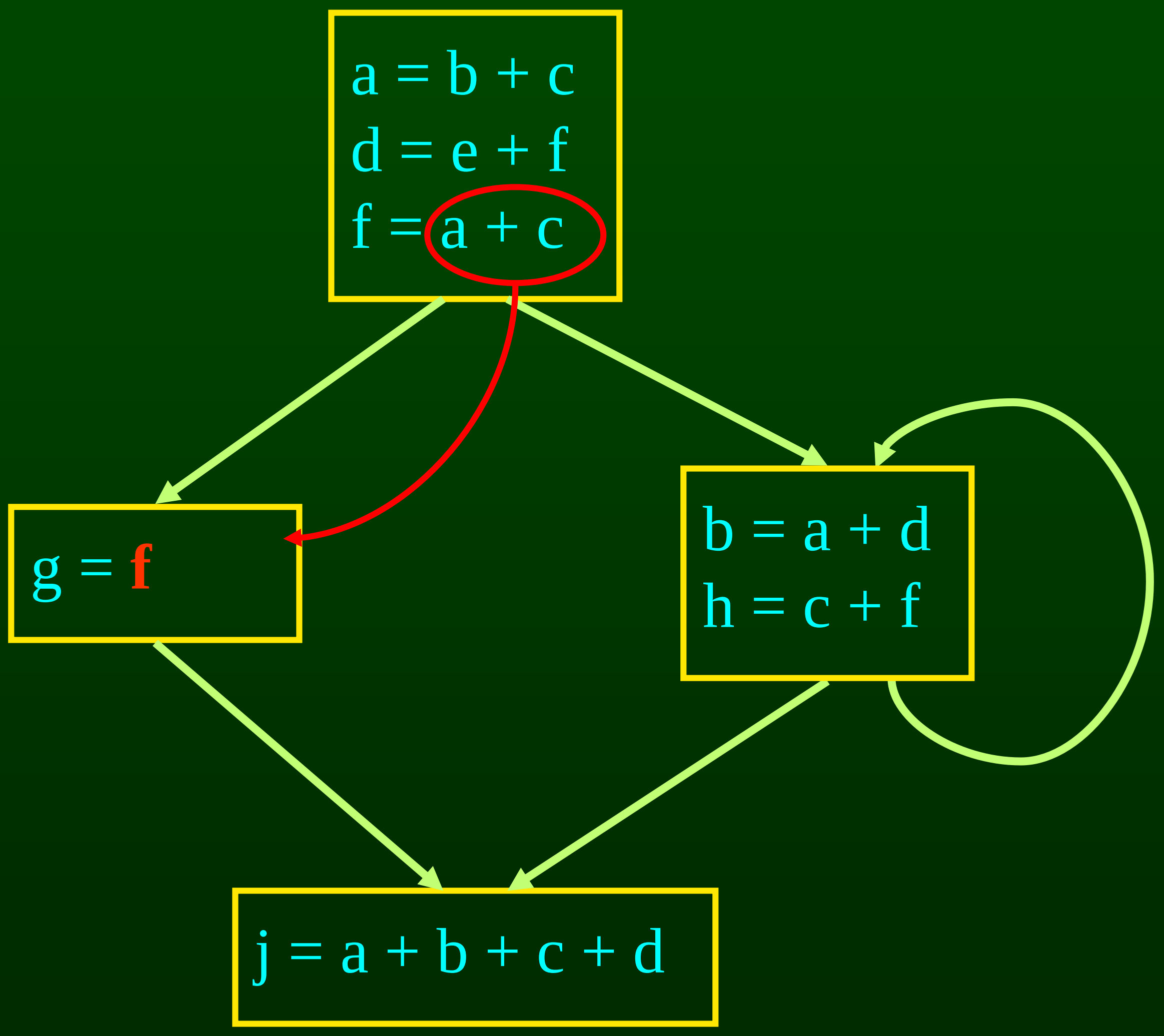
Use of Available Expressions



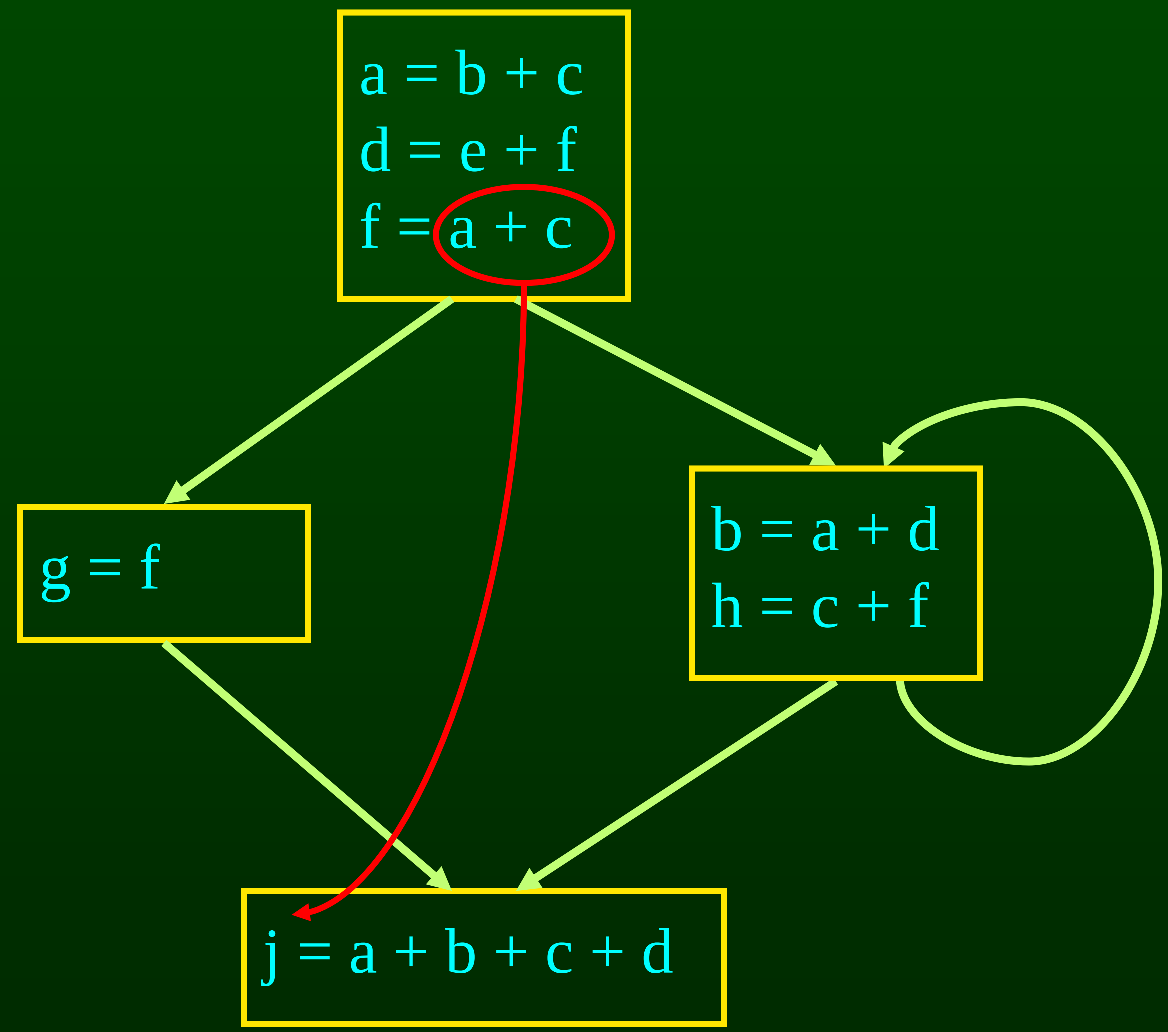
Use of Available Expressions



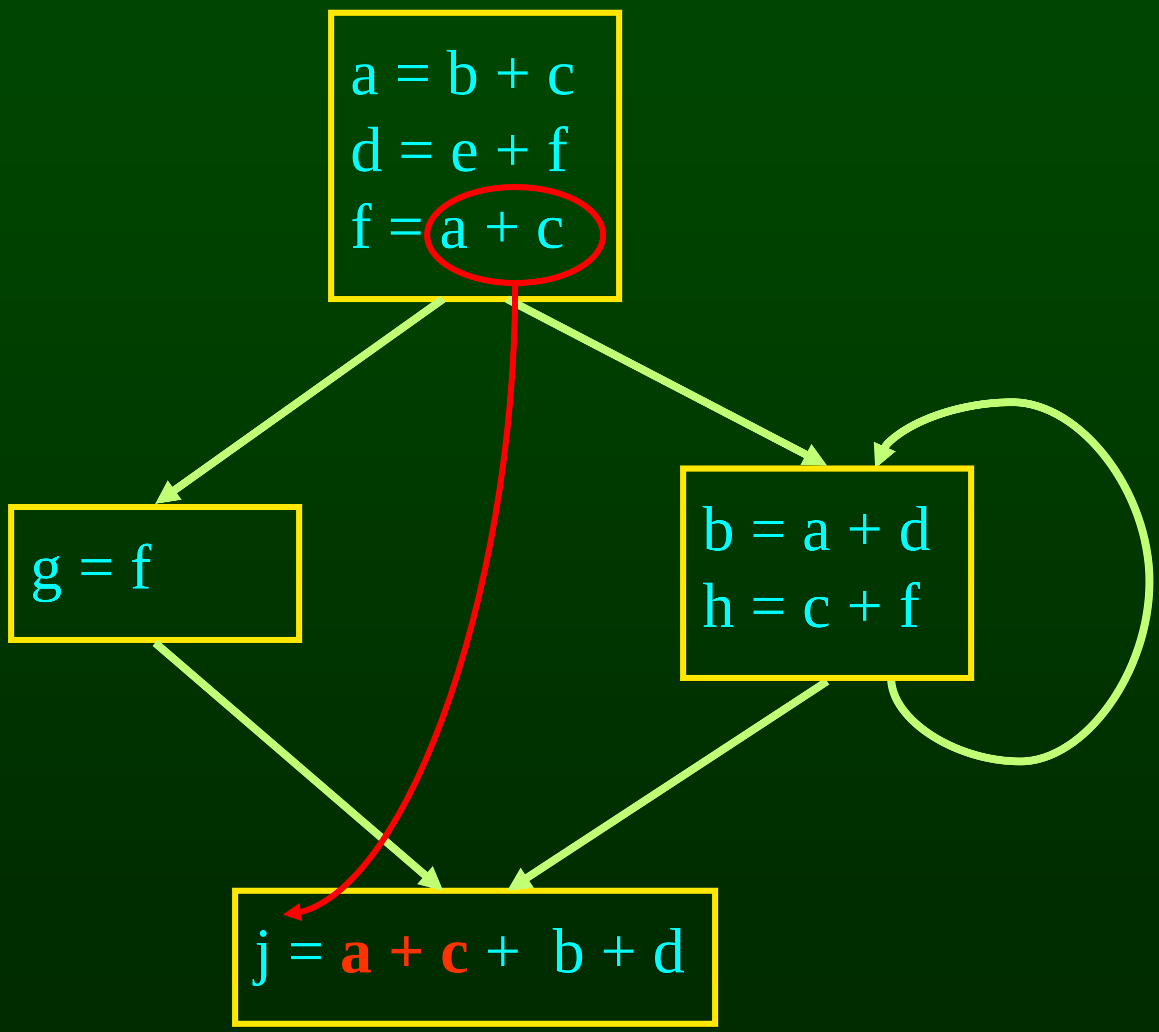
Use of Available Expressions



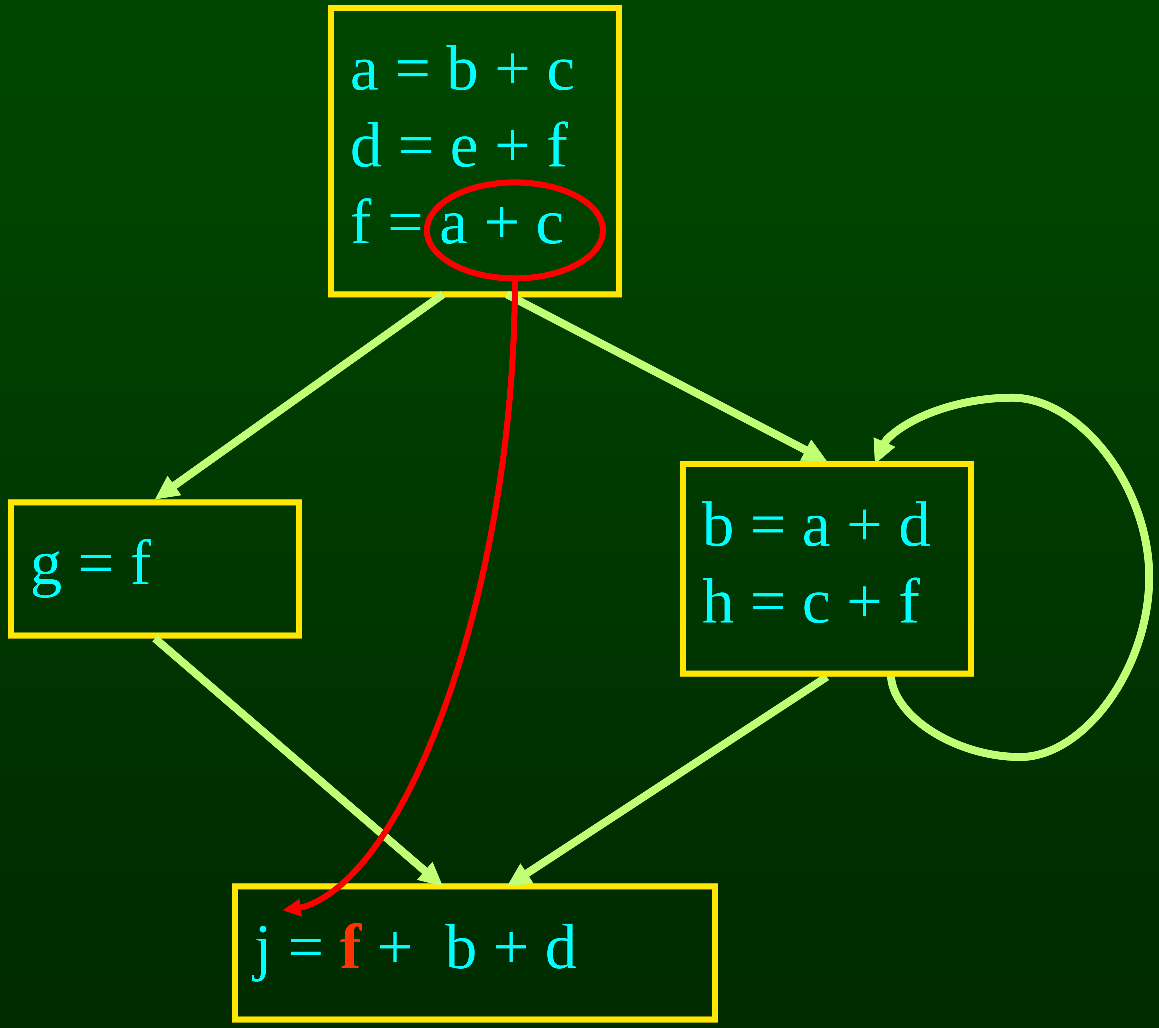
Use of Available Expressions



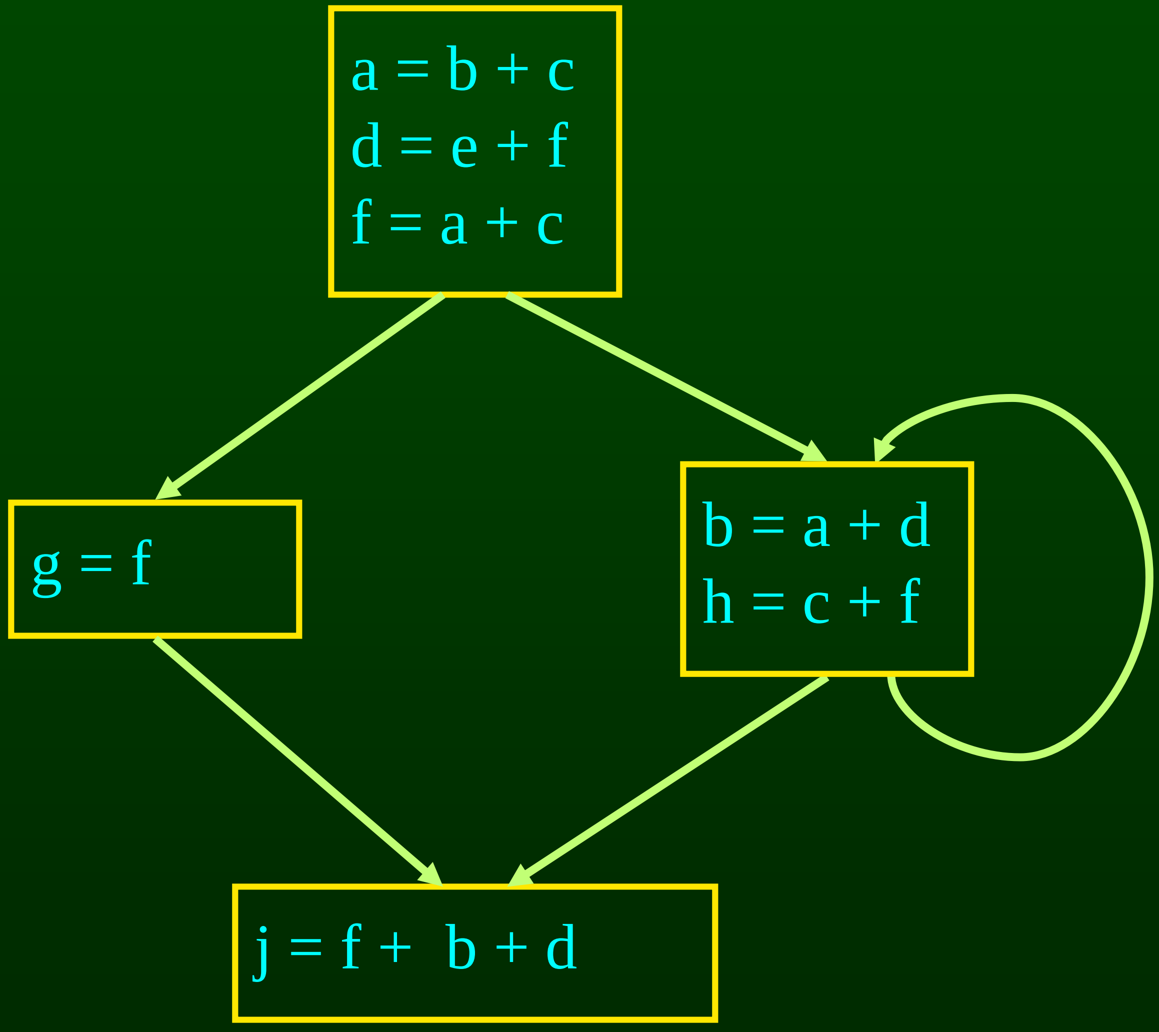
Use of Available Expressions



Use of Available Expressions



Use of Available Expressions



Computing Available Expressions

- Represent sets of expressions using bit vectors
- Each expression corresponds to a bit
- Run dataflow algorithm similar to reaching definitions
- Big difference
  - definition reaches a basic block if it comes from ANY predecessor in CFG
  - expression is available at a basic block only if it is available from ALL predecessors in CFG

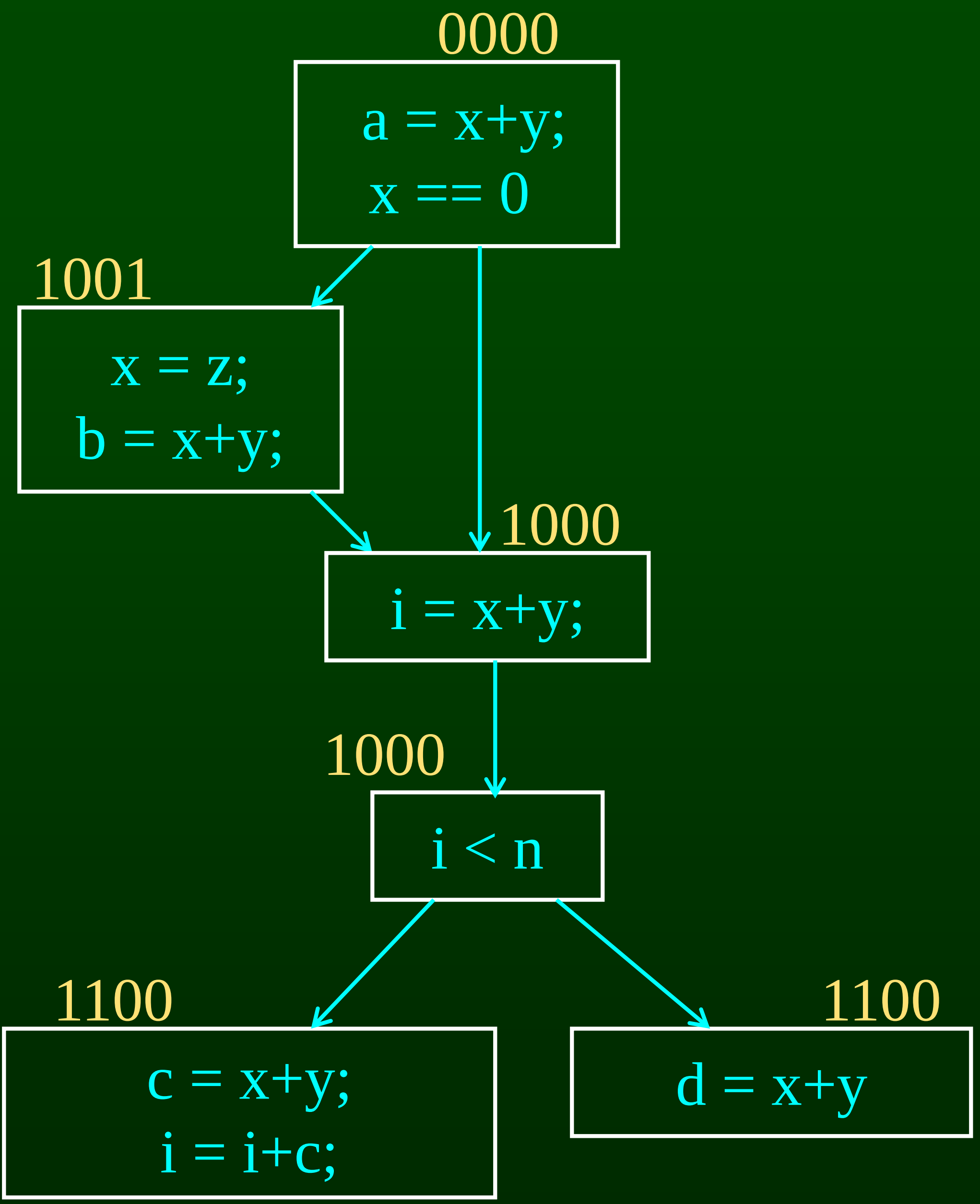
Expressions

1: x+y

2: i<n

3: i+c

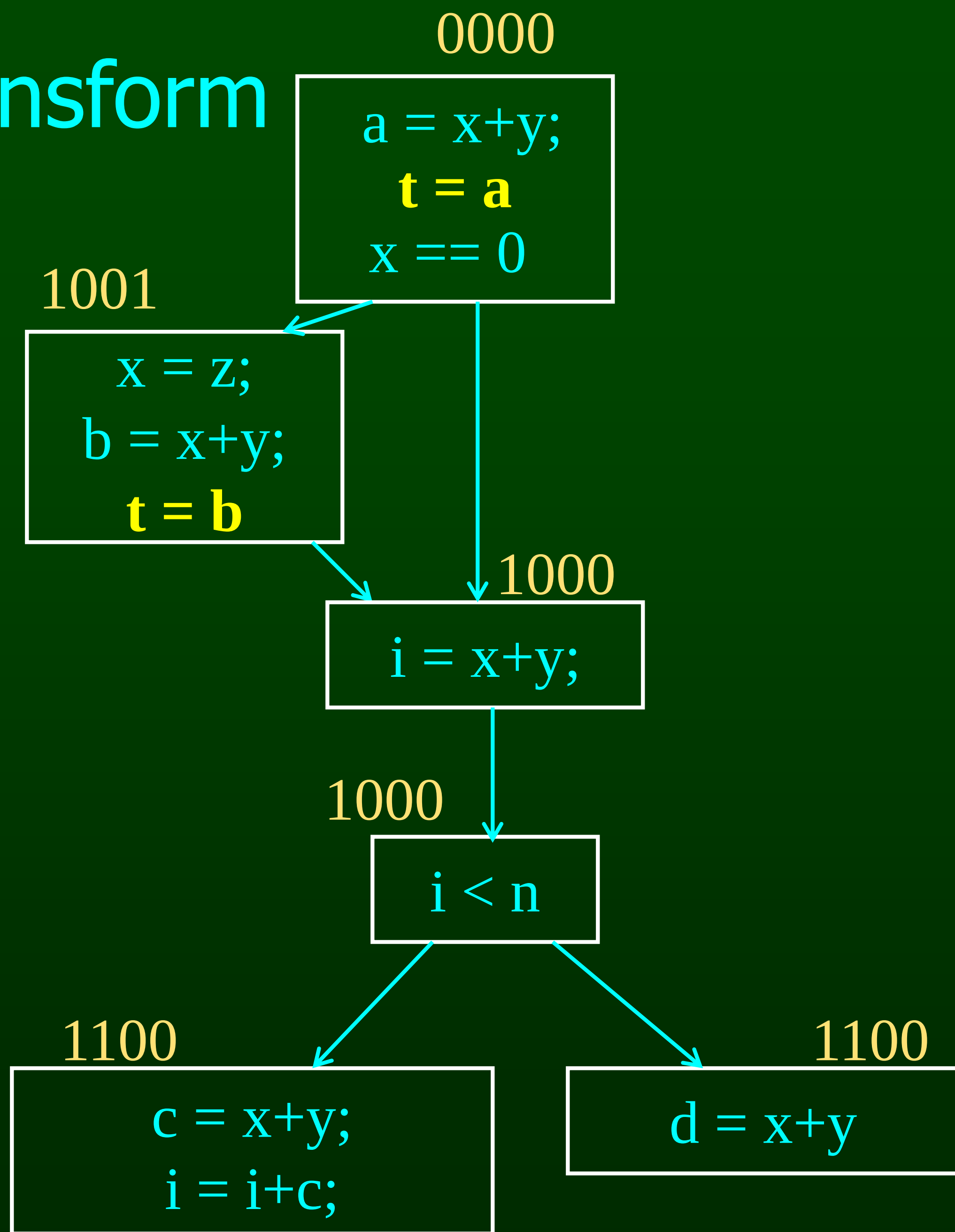
4: x==0



Global CSE Transform

Expressions
1: x+y
2: i<n
3: i+c
4: x==0

must use same temp
for CSE in all blocks



Global CSE Transform

Expressions

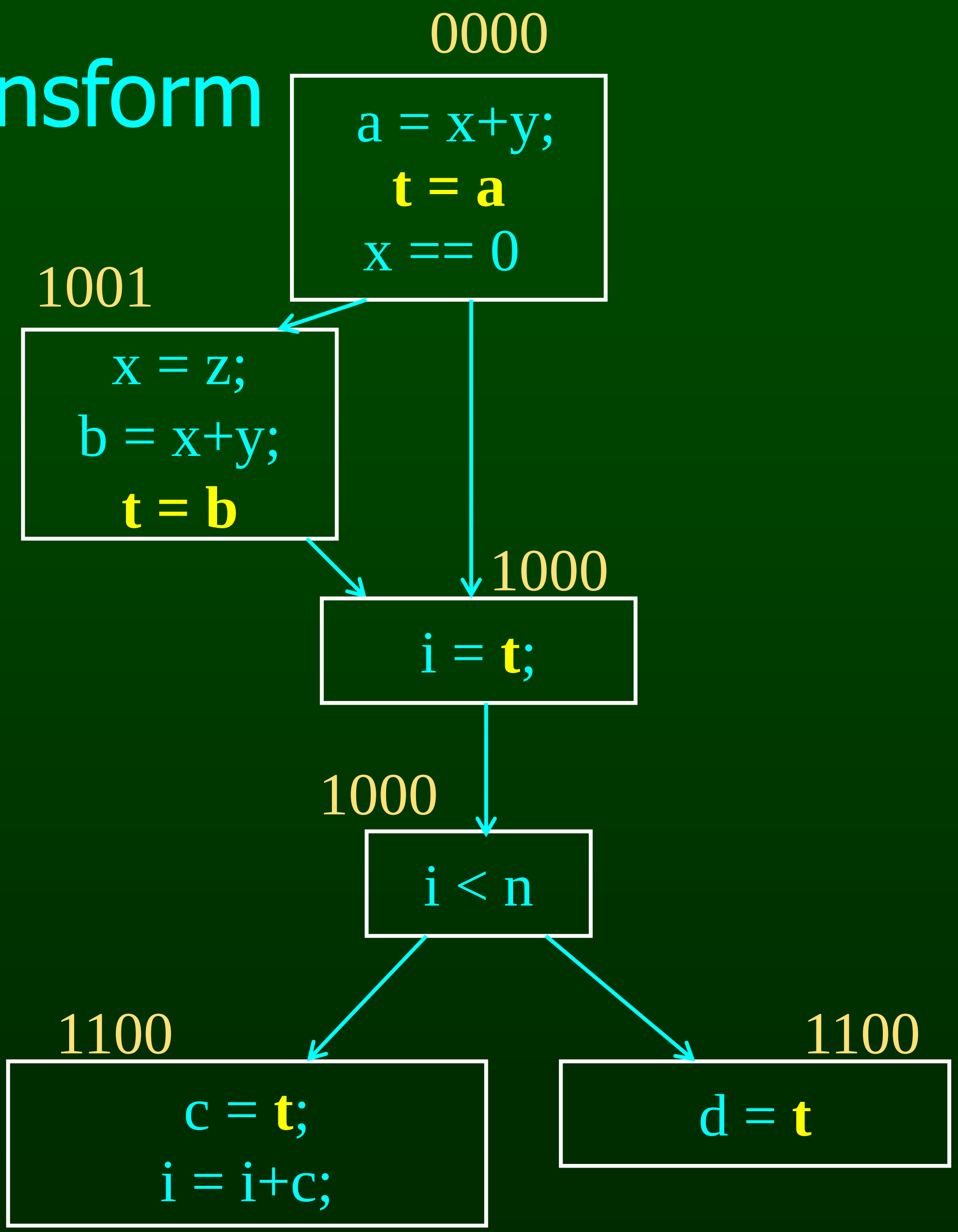
1: x+y

2: i<n

3: i+c

4: x==0

must use same temp for CSE in all blocks



Formalizing Analysis

- Each basic block has
  - IN - set of expressions available at start of block
  - OUT - set of expressions available at end of block
  - GEN - set of expressions computed in block
  - KILL - set of expressions killed in in block
- GEN[x = z; b = x+y] = 1000
- KILL[x = z; b = x+y] = 1001
- Compiler scans each basic block to derive GEN and KILL sets

Dataflow Equations

- $IN[b] = OUT[b1] \cap \ldots \cap OUT[bn]$
  - where b1, ..., bn are predecessors of b in CFG
- $OUT[b] = (IN[b] - KILL[b]) \cup GEN[b]$
- $IN[entry] = 0000$
- Result: system of equations

Solving Equations

- Use fixed point algorithm
- IN[entry] = 0000
- Initialize OUT[b] = 1111
- Repeatedly apply equations
  - IN[b] = OUT[b1] $\cap$ ... $\cap$ OUT[bn]
  - OUT[b] = (IN[b] - KILL[b]) U GEN[b]
- Use a worklist algorithm to reach fixed point

Available Expressions Algorithm

```
for all nodes n in N
      OUT[n] = E;  // E is set of all expressions
IN[Entry] = emptyset;
OUT[Entry] = GEN[Entry];
Changed = N - { Entry }; // N = all nodes in graph

while (Changed != emptyset)
      choose a node n in Changed;
      Changed = Changed - { n };

      IN[n] = E; // E is set of all expressions
      for all nodes p in predecessors(n)
            IN[n] = IN[n] ∩ OUT[p];

      OUT[n] = GEN[n] U (IN[n] - KILL[n]);

      if (OUT[n] changed)
            for all nodes s in successors(n)
                  Changed = Changed U { s };
```

Questions

- Does algorithm always halt?
- If expression is available in some execution, is it always marked as available in analysis?
- If expression is not available in some execution, can it be marked as available in analysis?

Duality In Two Algorithms

- Reaching definitions
  - Confluence operation is set union
  - OUT[b] initialized to empty set
- Available expressions
  - Confluence operation is set intersection
  - OUT[b] initialized to set of available expressions
- General framework for dataflow algorithms.
- Build parameterized dataflow analyzer once, use for all dataflow problems

Outline

- Reaching Definitions
- Available Expressions
- **Liveness**

Liveness Analysis

- A variable v is live at point p if
  - v is used along some path starting at p, and
  - no definition of v along the path before the use.
- When is a variable v dead at point p?
  - No use of v on any path from p to exit node, or
  - If all paths from p redefine v before using v.

What Use is Liveness Information?

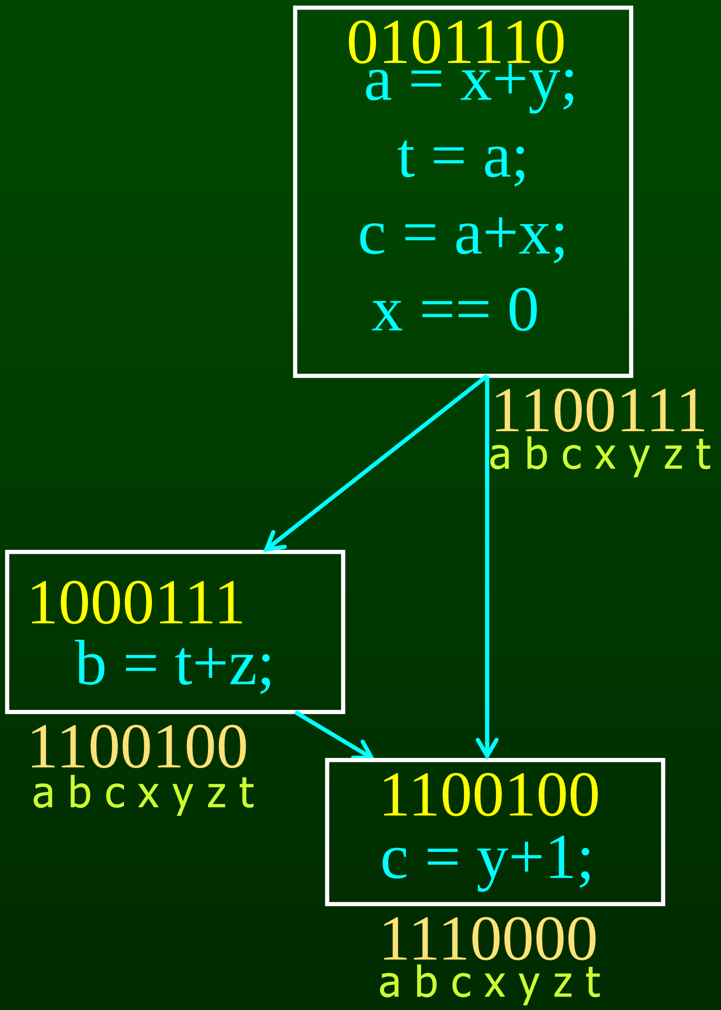
- Register allocation.
  - If a variable is dead, can reassign its register
- Dead code elimination.
  - Eliminate assignments to variables not read later.
  - But must not eliminate last assignment to variable (such as instance variable) visible outside CFG.
  - Can eliminate other dead assignments.
  - Handle by making all externally visible variables live on exit from CFG

Conceptual Idea of Analysis

- Simulate execution
- But start from exit and go backwards in CFG
- Compute liveness information from end to beginning of basic blocks

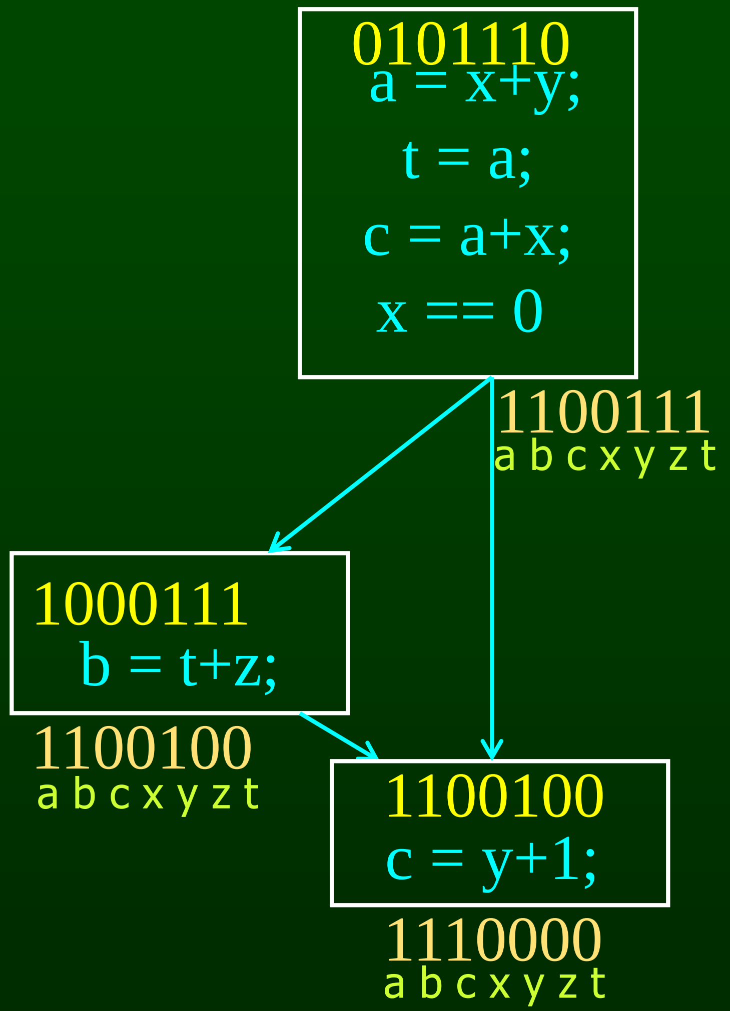
Liveness Example

- Assume a,b,c visible outside method
- So are live on exit
- Assume x,y,z,t not visible
- Represent Liveness Using Bit Vector
  - order is abcxyzt



Dead Code Elimination

- Assume a,b,c visible outside method
- So are live on exit
- Assume x,y,z,t not visible
- Represent Liveness Using Bit Vector
  - order is abcxyzt



Formalizing Analysis

- Each basic block has
  - IN - set of variables live at start of block
  - OUT - set of variables live at end of block
  - USE - set of variables with upwards exposed uses in block
  - DEF - set of variables defined in block
- USE[x = z; x = x+1;] = { z } (x not in USE)
- DEF[x = z; x = x+1;y = 1;] = {x, y}
- Compiler scans each basic block to derive USE and DEF sets

Algorithm

```
for all nodes n in N - { Exit }
      IN[n] = emptyset;
OUT[Exit] = emptyset;
IN[Exit] = use[Exit];
Changed = N - { Exit };

while (Changed != emptyset)
      choose a node n in Changed;
      Changed = Changed - { n };

      OUT[n] = emptyset;
      for all nodes s in successors(n)
            OUT[n] = OUT[n] U IN[p];

      IN[n] = use[n] U (out[n] - def[n]);

      if (IN[n] changed)
            for all nodes p in predecessors(n)
                  Changed = Changed U { p };
```

Similar to Other Dataflow Algorithms

- Backwards analysis, not forwards
- Still have transfer functions
- Still have confluence operators
- Can generalize framework to work for both forwards and backwards analyses

Comparison

Reaching Definitions

```
for all nodes n in N
    OUT[n] = emptyset;
IN[Entry] = emptyset;
OUT[Entry] = GEN[Entry];
Changed = N - { Entry };

while (Changed != emptyset)
    choose a node n in Changed;
    Changed = Changed - { n };

    IN[n] = emptyset;
    for all nodes p in predecessors(n)
        IN[n] = IN[n] U OUT[p];

    OUT[n] = GEN[n] U (IN[n] - KILL[n]);

    if (OUT[n] changed)
        for all nodes s in successors(n)
            Changed = Changed U { s };
```

Available Expressions

```
for all nodes n in N
    OUT[n] = E;
IN[Entry] = emptyset;
OUT[Entry] = GEN[Entry];
Changed = N - { Entry };

while (Changed != emptyset)
    choose a node n in Changed;
    Changed = Changed - { n };

    IN[n] = E;
    for all nodes p in predecessors(n)
        IN[n] = IN[n] ∩ OUT[p];

    OUT[n] = GEN[n] U (IN[n] - KILL[n]);

    if (OUT[n] changed)
            for all nodes s in successors(n)
            Changed = Changed U { s };
```

Liveness

```
for all nodes n in N - { Exit }
    IN[n] = emptyset;
OUT[Exit] = emptyset;
IN[Exit] = use[Exit];
Changed = N - { Exit };

while (Changed != emptyset)
    choose a node n in Changed;
    Changed = Changed - { n };

    OUT[n] = emptyset;
    for all nodes s in successors(n)
        OUT[n] = OUT[n] U IN[p];

    IN[n] = use[n] U (out[n] - def[n]);

    if (IN[n] changed)
        for all nodes p in predecessors(n)
            Changed = Changed U { p };
```

Comparison

Reaching Definitions

```
for all nodes n in N
    OUT[n] = emptyset;
IN[Entry] = emptyset;
OUT[Entry] = GEN[Entry];
Changed = N - { Entry };

while (Changed != emptyset)
    choose a node n in Changed;
    Changed = Changed - { n };

    IN[n] = emptyset;
    for all nodes p in predecessors(n)
        IN[n] = IN[n] U OUT[p];

    OUT[n] = GEN[n] U (IN[n] - KILL[n]);

    if (OUT[n] changed)
        for all nodes s in successors(n)
            Changed = Changed U { s };
```

Available Expressions

```
for all nodes n in N
    OUT[n] = E;
IN[Entry] = emptyset;
OUT[Entry] = GEN[Entry];
Changed = N - { Entry };

while (Changed != emptyset)
    choose a node n in Changed;
    Changed = Changed - { n };

    IN[n] = E;
    for all nodes p in predecessors(n)
        IN[n] = IN[n] ∩ OUT[p];

    OUT[n] = GEN[n] U (IN[n] - KILL[n]);

    if (OUT[n] changed)
            for all nodes s in successors(n)
            Changed = Changed U { s };
```

Comparison

Reaching Definitions

```
for all nodes n in N
    OUT[n] = emptyset;
IN[Entry] = emptyset;
OUT[Entry] = GEN[Entry];
Changed = N - { Entry };

while (Changed != emptyset)
    choose a node n in Changed;
    Changed = Changed - { n };

    IN[n] = emptyset;
    for all nodes p in predecessors(n)
        IN[n] = IN[n] U OUT[p];

    OUT[n] = GEN[n] U (IN[n] - KILL[n]);

    if (OUT[n] changed)
        for all nodes s in successors(n)
            Changed = Changed U { s };
```

Liveness

```
for all nodes n in N
    IN[n] = emptyset;
OUT[Exit] = emptyset;
IN[Exit] = use[Exit];
Changed = N - { Exit };

while (Changed != emptyset)
    choose a node n in Changed;
    Changed = Changed - { n };

    OUT[n] = emptyset;
    for all nodes s in successors(n)
        OUT[n] = OUT[n] U IN[p];

    IN[n] = use[n] U (out[n] - def[n]);

    if (IN[n] changed)
        for all nodes p in predecessors(n)
            Changed = Changed U { p };
```

Analysis Information Inside Basic Blocks

- One detail:
  - Given dataflow information at IN and OUT of node
  - Also need to compute information at each statement of basic block
  - Simple propagation algorithm usually works fine
  - Can be viewed as restricted case of dataflow analysis

Pessimistic vs. Optimistic Analyses

- Available expressions is optimistic (for common sub-expression elimination)
  - Assume expressions are available at start of analysis
  - Analysis eliminates all that are not available
  - Cannot stop analysis early and use current result
- Live variables is pessimistic (for dead code elimination)
  - Assume all variables are live at start of analysis
  - Analysis finds variables that are dead
  - Can stop analysis early and use current result
- Dataflow setup same for both analyses
- Optimism/pessimism depends on intended use

Summary

- Basic Blocks and Basic Block Optimizations
  - Copy and constant propagation
  - Common sub-expression elimination
  - Dead code elimination
- Dataflow Analysis
  - Control flow graph
  - IN[b], OUT[b], transfer functions, join points
- Paired analyses and transformations
  - Reaching definitions/constant propagation
  - Available expressions/common sub-expression elimination
  - Liveness analysis/Dead code elimination
- Stacked analysis and transformations work together